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Acting Chief, Army Security Agency

WAR DEPARTMENT office of the chief signal officer WASHINGTON

## ANALYSIS

 of A
## MECHANICO-ELECTRICAL CRYPTOGRAPH

PART I
$\nabla$

TECHNICAL PAPER
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## Section I

## INTRODUCTORY REMARKS

Nature of investigation tof results. | Par. | $\begin{array}{l}\text { Purpose of this paper_- } \\ 1\end{array}$ |
| ---: | :--- |
| 2 | Summary of conclusions | $\qquad$

$\qquad$ $\begin{array}{r}\text { Par. } \\ 3 \\ 4 \\ \hline\end{array}$ Preliminary statement of results

1. Nature of investigation.-In the latter part of 1923, a crytographic machine called the "Hebern Electric Super-Code" was submitted to the Chief Signal Officer for examination and consideration relative to its suitability for use in the military service. The usual claims for
 mer . This investign was
er mith a view to determining the merits of the device, more especially as to whether the degree of secrecy afforded by its use is sufficient to warrant further consideration as to its suitability for adoption in the military service.
2. Preliminary statement of results.-A cursory examination of the machine soon showed that it was worthy of the closest study. It is the smallest, most compact, and rugged device of its kind, considering the degree of secrecy which it is possible to achieve by its use. The latter
factor seemed to be considerably higher than that afforded by any other machine heretofore examined, excepting the Printing Telegraph Cipher Machine, which, in its present form, is much bulkier and not at all suitable for use in the theater of war below Army Headquarters. As a device for use in the field, the machine herein described seemed more nearly to fulfill the necessary requirements than any other machine ever studied by the writer
3. Purpose of this paper.-This paper was written for the purpose of setting forth in detail the results of the study of the cryptographic features of this machine. It is usually true that every really scientific system of cryptography presents a more or less unique case in crypt analysis, for the solution of which new principles and special methods of attack must be devised In this respect the system herein described does not lack novelty and interest for the crypt analyst, and if only for scientific and theoretical considerations that are involved in such a study, it has been thought worthy of being made the subject of detailed investigation and presentation. A preliminary knowledge of the more important and fundamental principles o cryptanalysis is necessary for a proper understanding of the technical details of this analysis. Reference is therefore made to signal Corps Traning Pamphlet No. 3, "El
, machine is but one of several recently patented cipher devices that are based upon very similar
${ }^{1}$ This paper was written early in 1924 , soon after the successful conclusion of the tests described in the ubsequent pages. Practically no changes, additions, or deletions have been made in the text as originally prepared.
cryptographic principles. The analysis herein presented is applicable to them, with minor modifications necessitated byslight differences in mechanical construction. With the ever increasing employment of radio telegraphy for military purposes, and the necessity for speedy, mechanical or electrical cryptographic apparatus, it is quite probable that machines of this nature will be used in future wars. They may, of course, be utilized by enemy military forces. A knowledge of the methods of analysis herein contained would be valuable and essential in the study of intercepted messages written by means of such devices. For this reason it has been deemed advisable to issue this paper as a secret document
4. Summary of conclusions.-It is shown in this paper that the machine under investigation, as at present constituted, produces cryptograms which are by no means "absolutely foirly high and the machine offers possibilities for modification with a wiew to augmenting the fairly high, and the machine offers possibilities for modification with a view to augmenting the degree of secrecy. One of its most serious disadvantages is that it makes no record of its opera-
tion, in the form of a printed copy of the dispatches enciphered or deciphered. It is understood tion, in the form of a printed copy of the dispatches enciphered or deciphered. It is understood
that the manufacturers are now engaged in producing a model which will make a printed record. that the manufacturers are now engaged in producing a model which will make a printed record.
If their efforts are successful, the new machine may be worthy of serious consideration for use in the military service.

Section II
DESCRIPTION OF MACHINE AND ITS OPERATION


Terminology Enciphering a dispatch Deciphering a dispatch Construction of cipher wheels
Function of bakelite separators--
The left and right fixed seguence
${ }^{\text {Par }}{ }_{5}$
Horizontal permutations of the cipher wheels Rutatory permutations of the cipher wheels Rermutations of LAW and RAW
8 Functions of LAW and RAW; automatic displace$\begin{aligned} & 9 \text { ment of the cipher wheels. } \\ & 10 \\ & \text { Potentialities of the machine }\end{aligned}$
5. Terminology.-For convenience in discussion, those parts of the machine which are essential to an understanding of this paper will be referred to under the following designations, which apply to Plate 1a

LAW-Left-hand aluminum wheel.
RAW-Right-hand aluminum wheel
CW1-First cipher wheel.
CW2-Second cipher wheel.
CW3-Third cipher wheel.
CW4-Fourth cipher wheel.
CW5-Fifth cipher wheel
E-D-Encipher-decipher set screw.
SET-Bench mark upon which the letters of LAW, the cipher wheels, and RAW are aligned in setting them according to the "key,"
BS1-Bakelite separator between LAW and CW 1
BS2-Bakelite separator between CW1 and CW2
BS4-Bakelite 2 arw
BS4-Bakelte separator between CW3 and CW4
BS6-Bakelite separator between CWF ad R1W
. Enciphering a dispatch.-To encipher a dispatch, the large knurled screw (E-D, plate 1a) at the right and towards the rear of the machine, is revolved so as to bring the indicator (on the top rear plate) to the left, to the position marked DIRECT. LAW, CW1 to 5, and RAW

 te sucessive letters of the plain test lisatel are depressed and the cipher letters that the sucess being illuninated on the lichtbord are written down. It will be noted that in
 movements will be diseused subsequently in full detail novements will be discussed subsequently in full detail.
7. Deciphering a dispatch-- Co decipher a dispatch, the enciphering-deciphering set screw of the key word, which must of course be known in advance REVERSE. Then the letter keys of the keyboard corresponding to the successive cipher letters are depressed, whereupon
their equivalent plain-text letters will be illuminated on the lightboard. It is to be stated that while the normal method of encipherment and decipherment is with the indicator set at DIRECT and REVERSE, respectively, this is not absolutely essential. The machine will encipher and decipher just as well with the opposite arrangement, i.e., set to REVERSE for encipherment and to DIRECT for decipherment, but the correspondents must, of course, be in agreement in this respect.
8. Const
8. Construction of cipher wheels.-Plate 1b shows one of the cipher wheels, which are all similar in construction. The rim of the wheel is divided up into 26 equal sections, hereafte designated as cipher-wheel segments, which are labeled by means of the normal alphabet, Z being eplen tion serments are in reces into which a lever may fall and thus cause the wheel to be displaced one step at a time. Each peripherally lettered cipher-wheel seament has two electrical contact surfaces on the sides or faces of the wheel, on on the left, hereafter designated as the left-hand contact, abbreviated as LHC, and one on the right, similarly designated as the right-hand contact, abbreviated as RHC. Each LHC is provided with a binding post on the left face of the wheel, and each RHC with a binding post on provided with a binding post on the left face of the wheel, and each RHC with a binding post on
the right face. An insulated conductor connected to the binding post of the LHC of every cipher-wheel segment goes through a hole in the center plate of the wheel, and is connected to the binding post of the RHC of some other segment on the same wheel. Thus, for example, on CW1 the LHC of A is connected to the RHC of G; the LHC of E is connected to the RHC of O the LHC of K is connected to the RHC of Z , as may be shown diagrammatically thus:


The series of LHCs of all the segments are connected to the series of RHCs in an arbitrary mixed order. The set of connections established in this manner is different in each cipher wheel. These cipher wheels, therefore, act merely in the capacity of different mixed alphabets, which may be indicated diagrammatically in the alphabets below

ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWYYZ
Alphabet 1.
BDFCWPAOMSHXVIEUQZYGJTNLRKBDFCWPAOMSHXVIEUQZYGJTNLRK
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKL MNOPQRSTUVWXYZ WYDHKPUQAJFOTCMIVZSEGLNRXBWYDHKPUQAJFOTCMIVZSEGLNRXB ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ HLSZVDJNXBTFMRPAWOUGIQECKYHLSZVDJNXBTFMRPAWOUGIQECKY

ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ DKHXWAFYORVBIMPTJEUQSCGLNZDKHXWAFYORVBIMPTJEUQSCGLNZ
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ

Alphabet 2.
Alphabet 3.
Alphabet 4.
Alphabet 5.

In the case of the alphabet applying to each cipher wheel, it is to be understood from what has preceded, that a wire not shown in the diagrammatic representation actually connects each etter in the upper sequence of letters to the same letter in the lower sequence. Thus, for RHC $G$ ?


corresponding to the letter G. The upper sequence of letters in each cipher alphabet, because it coincides with the normal or standard alphabet, will be referred to hereafter as the normal component; the lower sequence in each alphabet, as the mixed componen. The normal compo nent, therefore, corresponds to and indicates the sequence of LHCs; the mixed component corresponds to the sequence of RHCs and indicates the point from which a current entering a LHC will emerge on the right hand side of the cipher wheel. For purposes of abbreviation, an alphabet will hereafter be designated by the letters AL followed by a number. Thus, AL1 refers to Alphabet 1. The normal component of AL1 will be designated by the abbreviation NAL1; the mixed component, by MAL1. Corresponding abbreviations will apply in the case of the other alphabets.
9. Function of bakelite separators.--Four circles of 26 fixed, spring-contacts, which ar in the bakelite separators designated as BS2 to BS5 in Plate 1c, and which can only be seen by removing the cipher wheels, serve to form fixed paths for conducting the current from the current morely bing earied directly across the bokelite disk. For evomple, with CIW CW Cher both a
 enter CW2 at the LHC of segment A; or in the reverse
CW2 at the LHC of A will enter CW1 at the RHC of A
10. The left and right sequences.-When the machine is set for DIRECT operation, the bakelite separators BS1 and BS6 contain the sets of contacts connected to the keyboard and lightboard contacts, respectively. The connections are not made directly, but through the intermediacy of a bakelite switching plate in the rear of the machine. The function of this plate and the nature of the connections there established will be described later (Section XIV). Suffice it to indicate at this point that the connections are established in such a manner as to produce the equivalents of two mixed alphabets corresponding to the sequences of the contacts in BS 1 and BS6. For example, with the indicator set at DIRECT, starting with the first contact point of BS1 on a line with SET, this contact point is connected to the key " $B$ " of the keyboard, and the homologous one in BS6 is connected to the lamp illuminating the letter "T". The next one, proceeding toward the rear of the machine is connected to the "S" of the keyboard, and " Y " of the lightboard, and so on, according to the following sequences, hereafter designated as the left fixed sequence, LFS, and the right fixed sequence, RFS, respectively:

With the indicator set for DIRECT operation the keyboard contacts lead to the contacts in BS1 (after passing through the rear plate mentioned above), and therefore LFS is applicable in BS1 (after passing through the rear plate mentioned above), and therefore LFS is applicable ou them, the lightboard lamps are connected to BS6 (after passing through the rear plate), and ections are reversed, the keyboard connections being through RFS, and the lightboard connections through LFS
11. Horizontal permutations of the cipher wheels.-The cipher wheels being identical so ar as their physical construction is concerned, they are all interchangeable, and any wheel can be inserted in any of the five positions on the shaft. There may be any number of cipher wheels rom which a selection of five different ones can be made. Having selected a set of five cipher wheels to be used, since each one of them can occupy the first, second, third, fourth, or fifth position, the number of different arrangements or permutations of the cipher wheels themselves, as regards their relative order upon the shaft from left to right, hereafter termed the horizontal permutations of the cipher wheels, is $5 \times 4 \times 3 \times 2 \times 1$, or 120 .

But the cipher wheels may also be mounted upon the shaft in an "upside down" position, that is, so that what were previoisly the RHCs now become the LHC's, and vice versa; this procedure, as will appear subsequently, vields an entirely new series of equivalents. Thus, each cipher wheel may be regarded as being the equivalent of two wheels. Therefore the five cipher wheels really amount to $5 \times 2$, or ten wheels. Now there are five positions in which wheels can be inserted either right side up or upside down. The first insertion can be made in any one of the five positions, $1,2,3,4$, or 5 . There being ten wheels, for the first insertion there are ten possibilities. Having inserted one cipher wheel in position there are four positions left, and any one of eight wheels can be inscrted, yielding eight possibilities. The third insertion yields six possibilities, the fourth, four, and the fifth, two possibilitics. Therefore, the total number of possible horizontal permutations of the five cipher wheels themeelyes, as regards their relative order upon the shaft, is $10 \times 8 \times 6 \times 4 \times 2$, or 3,840 . If there were more wheels available, fron which a set of five were to be selected, the number of permutations on the shaft would be stil reater, according to the formula

$$
\mathbf{N}=2 n \times 2(n-1) \times 2(n-2) \times 2(n-3) \times 2(n-4)
$$

where $\mathbf{N}$ is the total number of horizontal permutations, and $n$ is the total number of cipher wheels from which a set of five can be selected.

Each one of these permutations or horizontal arrangements of the cipher wheels upon the shaft will yield different results in encipherment so that, for purposes of communication, it becomes absolutely essential to know exactly which horizontal permutation is in effect at any given moment.
12. Rotatory permutations of the cipher wheels.-When mounted upon the shaft, each cipher wheel is susceptible of being placed in any one of 26 different positions relative to the etter of its periphery which is aligned on the bench mark or setting line, SET. When two cipher wheels are inserted, they are susceptible of being placed in any one of $26 \times 26$, or 676 different positions relative to the pair of letters which are aligned on SET. When all five wheels are inserted, they are susceptible of being placed in any one of $26^{5}$, or $11,881,376$ different positions ehative to the set of five letters which are aligned on SEI. Each diferent alignment of the Ave cipher wheels from left to right on the shaft will hereafter be referred to as one of the rotalory permutations of the cipher wheels. It is obvious that for every one of the rotatory permutation the complete electrical path established for the passage of an electrical current from a given contact of BS1, through the ciplier wheels and to a given contact of BS6 is different, considered as a whole. That is, the five cipher wheels provide a total of $11,881,376$ different complete paths for the progress of the current from each contact of BS1 through the five cipher wheels into a contact of BS6. Since there are 26 contacts in BS 1 , one for each letter of the alphabet, it follows that for any given rotatory permutation of the cipher wheels there exists a different, or unique secondary cipher aphabet; and since there are 1, 081,376 difcrent rotatory permutations, it iol lows the the
is eerned directly in the electrical relations, but are vitally concerned in the mechanical rolations en olso asume diffent notatory positions upon the shaft. Since there are but two whels, and they are not ine of the pair of them, desionated by the 676 nermutations of the letiers of the alphabet taken in pairs. These designations form a part of the key word, the first and last letters of the word being used to determine the initial positions of LAW and RATV, respectively. Considered collectively, or as a unit, the particular horizontal and rotatory permatation of the cipher wheels, and the particular rotatory permutation of LAW and RAW in effect during the encipherment
of a letter of the plain text, or the decipherment of a letter of the cipher text, constitute the key", in the cryptographic sense of the term.
14. Functions of LAW and RAW; automatic displacement of the cipher wheels.-These two wheels are the principal agents in controlling the automatic displacements or motions of two of the cipher wheels, viz., CW1, and CW3. RAW controls CW1, and LAW, CW3. The principal feature of this control, so far as this analysis is concerned is explained in connection with the accompanying sketch, fig. 1

Depression of any key causes a universal bar, LBB, to rotate a rocker shaft 1, attached to which are four levers or wheel-stepping "dogs." Dog 2, the lower end of which falls into one of the 26 small notches on the right hand side of RAW, serves to move RAW one step forward per


CuB

## Figure 1-Diseram of mechamieat action on cipher whecls, LAW and RAW.

depression of any key; another dog, 3, does the same thing with respect to CW5. Since CW5 and RAV can be advanced only 26 times in making one complete revolution, their period in terms of letters is 26 . Dogs 4 and 5, also attached to shaft 1 cannot move CW 1 and CW3 be cause the arms 6 and 7 are so placed as to prevent the ends of these two dogs from falling into the recesses of CW1 and CW3.

Now when trip dog 8 falls into the single large notch on the left hand side of RAW (at letter Z on this wheel, and when $\mathbf{N}$ is at SET) shaft 9 is caused to rock. At the other end of shaft 9 there is a release arm 10, which allows dog 11 to drop into one of the 26 small notches on the telt hand side of LAW. Dog 11 is attached to a sleeve 12, which swings freely on shaft 1. At the ther end of sleeve 12, release arm 6, mentioned above, moves when dog 11 drops into a notch on depression of a key causes both LAW and CW1 to advance one step. Since dog 8 falls into the

Z notch of RAW but once per revolution, i.e., once per 26 letters, and since release arm 10 thus allows dog 11 to fall into one of the notches of LAW but once per 26 letters, the period of LAW is $26 \times 26$ or 676 letters. Likewise, the period of CW1 is 676 letters

Now dog 13 drops into the single large notch at $Z$ on the right hand side of LAW only once per revolution of LAW. This dog 13, is attached to a sleeve 14 on shaft 9 , and is free to swing on the shaft. At the other end of sleeve 14, arm 7, vibich normally prevents dog 5 fron dropping into a recess on CW3, is withdrawn and allows dog 5 to drop whenever dog 13 drops. This happens but once in 676 letters, and thus CW3 is advanced one step per 676 depressions The period of CW3 is therefore $670 \times 26$ or 17,576 letters

In a previous model of the machine CW4 and CW5 were caused to advance by a similar arrangement of dogs and release arms. This, howerer, was thought unnecessary by the manufacturers, ard in the model studied these two wheels could only be adranced manually.

It will be convenient to designate the letter $O$ as the finishing point because the displace plete period of LAW and RAW, and the letter , as the finishing point, because the
ments of CW1 and CW3 occur when N is at SET and a key is just then being
The displacement relations of the wheels may be summarized as follows:
(1) CW5 and RAW are displaced one interval per depression of any key. Their period is 26
letters. (2) CW 1 and . The periods of CTir and LAW are therefore 676 letters
(3) CW3 is displaced one interval per one complete revolution or period of LAW, and hence its period is 17,576 letters. CW2 and CTW4 do not unde
and unless moved by hand remain fixed in their positions.
The length of the period produced by automatic displacement of the cipher wheels is 17,576 letters, but by displacing CW2 and CW4 by hand, the lengtlo of the periou can be increased to $17,576 \times 676$, or $11,881,376$ letters. That is, if each correspondent used a different permutation of CW2 and CW4, then there is possible a series of 676 direrent periods, each of 17,566 letters, and a total of $11,881,376$ letters could be enciphered without repectition of cipher alphabets
15. Potentialities of the machine.-It is absolutely essential for a full understanding of the subsequent analysis that a clear conception be had of the really staggering number of possible permutations and combinations afforded by the machine, and thas can best be set forth in terms of the number of different paths that are available for an electric current to take in the process of encipherment of a single letter

The exact path travcrsed is determined by a particular combination of the following six factors
(1) The plain-text letter that is being enciphered.
(2) The position in thie left fixed sequence, LFS, that is determined by the keyboard con(2) The position in the left fixed sequence, LFS, that is de
(3) The position in the right fixed sequence, RFS, that is determined by the lightboard contact of the cipher letter that is to result from the encipherment
(4) The setting of the machine with respect to the direct and reverse method of operation.
(5) The horizontal permutation of the cipher wheels that is in effect during the encipherment
of the letter
6) The rotatory $p$ of the letter.
Consider now what happens when any key of the keyboard is depressed. A connection is established from the battery, through the contacts of the depressed key, through the rear plate (see paragraph 10), to some contact in the LFS, from which it emerges at some contact in BS1.

This current can enter CW1 through any one of its 26 LHC 's, depending upon the rotatory position of CW1. Now having entered CW1 through a certain one of its 26 LHC's, the current will leave this cipher wheel through a certain one of its 26 RHC's, the particular one being determined only by the internal wiring of CW1 as explained in paragraph 8. But so far as any given letter of CW1 is concerned, the path followed by a current which has entered a given LHC is always the same on that cipher wheel, so long as the wiring is unchanged in the wheel. On leaving CW1 the current is carried directly across BS2 and enters one of the 26 LHC s of CW 2 , the particular one bet
 five CIW's are ensided a $11,881,376$ paths in finding its exit at a RHC of CIF takes being determined by a fived set of conditions as coved by the rotatory permuthong the five wheels as they are mounted according to a riven horizontal permutation upo the shaft. Now assuming that the five cipher wheels were caused to be displaced in such apon the shaft. present successively to the SET line every one of the possible $11,881,376$ rotatory permutations, it will follow that, given an initial setting of the five wheels the particular permutation of the five separate paths composing the complete path traversed by a current caused by the first depression of a single key cannot be exactly the same for the succeeding $11,881,375$ depressions of the same key, but will be the same for the $11,881,370 \mathrm{th}$ depression after the first.

Now was shown abore that there are 3,840 horizonta arangements of five cipher whees, and since each one of them can yield $11,881,376$ retatory permutations, the total number of this which a single set of five cipher whicels can provide is $3,840 \times 11,881,366$, or $45,624,483,840$. Furthermore, since the machine, when the indicator is set to RETEREE, will function just as efficiently for enciphering purposes as it will when it is set to DIRECT (sce paragraph 7), and since in the former case the cipher equivalents are altogether difierent from what they are in the atter case, a second series of $45,624,483,840$ pathis is possible. That is, the cipher equivalent for a given letter may be the result of the traversing of a current of electricity over any one of over 91 billion different circuits from a key of the keyboard to a lamip of the lightboard, as provided by this small machine with but five cipher wheels.

The final cipher equivalent, however, in any case is expressible as, and can be but one of, 26 characters. Hence there will obviously be a myriad of repetitions of cipher letters brought about by this truly staggering number of different patis, but the order of repetition will be in an pparmined by Whecls. What has and the toll the leys so that may aid the lphabets are involved in this systom. Anothor way of statior the case is this. By riliciper the possibilities of one set of cipher whels on mache it would posible and and series of dispatches concisting of over 01 billion letters before encipherment by ilentical sequence of alphabets would becin and thus produce two or more messares in the same "trey," For reasons which will become apparent subsequently, this analysis will for the present b estricted to that method of using the machine in which the only variable factor concerned in the keys for a set of dispatches is that involving chances in the initial rotatory permutation for each dispatch. It will be assumed (1) that this initial permutation is to be given as the first word in the text of each dispatch; (2) that the machines are to be set at DIRECT for encipher ment and at REVERSE for decipherment; and (3) that no change is made in the horizontal permutation of the cipher whecls during the course of the encipherment of the set of dispatches o be subjected to analysis.

Section III

## BASIC CRYPTOGRAPHIC PRINCIPLES OF OPERATION

## Cipher relations

Use of sliding alphe-..............................Enciphering an example Deciphering by means of sliding alo................ Par.
18
19 traversed in enciphernent.
16. Cipher relations.-Having described the mechanics of the machine, and the manner in which the depression of a key of the keyboard causes a circuit to be established which results in the illumination of a lamp of the lightboard, the cryptographic principles of the system will now be examined more in detail.

The best way to do this is first to determine exactly how a letter is enciphered (from the cryptographic standpoint, not the mechanico-electrical); then determine what, if any, are the relations between successive encipherments of the same letter; and finally, determine what, if any, are the relations between encipherments of dissimilar letters. A set of sliding strips coinciding with the alphabets of the machine will be used for this purpose and it is recommended that the reader provide himsel win are arranged in th in illustration.

## SET

BSXRZTKDNGCHMVOLYQEUPWJAIF. $\qquad$ Left fixed Left fixed
sequence

ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ ABCDCWPAOMSHXVIEUQZYGJTNLRKBDFCWPAOMSHXVIEUQZYGJTNLRK Alphabet 1
$\left.\begin{array}{l}\text { ABCDEFGHIJKLMINOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ } \\ \text { WYDHKPUQAJFOTCMIVZSEGLNRXBWYDHKUQAJFOTCMIVSEGLNRXB }\end{array}\right\}$ Alphabet 2 (abcor ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ
HLSZVDJNXBTFMRPAWOUGIQECKYHLSZVDJNXBTFMRPAWOUGTQECKY Alphabet 3
$\left.\begin{array}{l}\text { ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ } \\ \text { DKIIXWAFYORVBIMPTJEUQSCGLNZDKHXWAFYORVBIMPT,IFITRSCGT NZ. }\end{array}\right\}$ Alphabet 4
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ $\}$ Alphabet 5 FRISYADPLJUXZGKOBTWCVMHEQNFRISYADPLJUXZGKOBTWCVMHEQN

TYOEUMXDFJQVKWBNSHCILRZAGP. $\qquad$ Right fixed
sequence.

SET
These strips are to be arranged so that the left and right fixed sequences are held in place by thumb tacks on a drawing board, and between these two strips the cipher alphabets should slide freely. The arrow coincides with the setting line SET, and it will be noted that the first letter of the left fixed sequence is directly over the first letter of the right fixed sequence. By using
these strips it is possible to duplicate the results of the machine in every detail. Instead of the mere depression of a key that closes a circuit from the positive pole of battery, through a keyboard contact, a contact in BS1, a path established by the cipher wheels, a contact in BS6, a lamp, and thence to the negative pole of the battery, the mind of the cryptanalyst must perform an equivalent function through the agency of the strips of alphabets; and whereas the electric ile er me the of 180,000 miles per sech, an at an and ver present risk of inaccurac.

Set the machine ${ }^{1}$ to the following keyword:

$$
\begin{array}{cccccccc}
\text { Wheels.-.- } & \text { LAW } & \text { CW1 } & \text { CW2 } & \text { CW3 } & \text { CW4 } & \text { CW5 } & \text { RAW } \\
\text { Setting }-\ldots-- & \text { S } & \text { I } & \text { G } & \text { I }
\end{array}
$$

and depress $A$ on the keyboard. The cipher resultant, as illuminated on the lightboard, is P, or $A_{\mathrm{D}}=\mathrm{P}_{\mathrm{c}}$. It will be noted that the circuit is not established until after the wheels CW5 and RAW have been advanced by the depression of the key to their next positions. In other words, the setting by means of which a letter is enciphered is shown only after the encipherment has been effected. This is very important to keep in mind. In the case just noted, with the setting SIGNALS, the actual setting at which encipherment was effected is SIGNAMT, in which it will be noted that CW5 and RAW have advanced one step. Hence, hereafter, that setting of the wheels which governs the actual path taken by the current during the encipherment of a letter will be termed the effective setting, and will be the one immediately following the apparent or keyword setting.

In setting up the wheels to a keyword, it is obvious that the setting letters on the cipher wheels correspond only to the LHC designations, and not to the RHC designations of the wheels, becuse the seque of letcrs on the periphery of cach cipher whee is the normal athabet sequembered in setting the sliding strips an a
17 Use of sliding alphabets in tracing electrical paths traversed in encipherment.-T dentical result will now be found by means of the sliding strips. First, it is necessary to set the dentical resnlt will now be found by means of the sidin.e strips. First, it is necessary to set the strips in reative positions corresponding to the relative positions of the cipher wheels, viz., in
the order LFS-AL1-AL2-AL3-AL4-AL5-RFS, and these must be juxtaposed relative to their the order LFSS-AL1-AL2-AL3-AL4-AL5-RFS, and these must be juxtaposed relative to their Although the wheels LAW and RAW do not enter into the electrical relations, and may be disregarded in tracing actual paths through the sliding strips, they must, nevertheless, be kept in mind constantly, as will be discussed later in connection with the displacement of AL1 and AL3. Now since the second lel ter of the heyword is the one that governs the position of CW1 and since this letter in the case of the keyword SIGNALS is I, therefore AL1 is set so that I of its normal component is under the setting arrow; similarly AL2 is set so that G of its normal component is under the setting arrow; ALS is set so that N of its normal component is under the setting arrow; AL4 is set so that $A$ of its normal component is under the setting arrow; and U 110
assumed that the reader
that indicated in the preceding pages. As etated above, actual possession of a machine is not absolutely essential, inasmuch as every function of the machine ean be duplicated by cmploying a set of sliding alphabets.

AL5 is set so that M of its normal component (to correspond with the effective setting SIGNAMT) is under the setting arrow. All of these settings are as indicated in the following diagram:

## SET

STUVWXYZABCDEFGHIJKLMNOPQR LAW BSXRZTKDNGCHMVOLYQEUPWJAIF

ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGH BDFCWPAOMMSHXVIEUQZYGJTNLRKBDFCWPAO. ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEF
WYDHKPUQAJFOTCMIVZSEGLNRXBWYDHKP
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLM HLSZVDJNXBTFMRिPAWOUGIQECKYHLSZVDJNXBTFM

ABCDEFGHIJKLMNOPQRSTUVWXYZ.
DKHXWFYORVBIMPTJEUQSCGLNZ
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKL ABCDEFGHIJKLMNOPQRSTUMWXYZABCDEFGHIJKL.

> TYOEUMXDFJQVKWBNSHCILRZAGP $\vdots$ $\frac{T}{\uparrow} U W W X Y Z A B C D E F G H I J K L M N O P Q R S ~$ SET

AL5
\} AL1
\} AL2

The strips are now ready to serve as guides in encipherment. In employing the machine itself, it was found that with the keyword given, $A_{p}$ was enciphered as $P_{c}$. Refer now to the sliding strips. Find A in LFS; it is the 24th letter of the sequence and is directly over F o NAL1. Imagine a wire connecting the F of NAL1 (which, it will be recalled, represents the set of LHC's of CW1) to the F of MAL1. It will be found that F of MAL1 is directly under C of NAL1; that is, the current originating at $\Lambda$ of LFS, entering CW1 at the LHC of $F$, emerges from CW1 at the RHC of C. As shown in the diagram of alphabets above, C of NAL1 is now opposite A of NAL2. Imagine a wire connecting A of NAL2 to A of MAL2. The latter will be found under I of NAL2 (that is, the RHC of I of CW2), and I is now opposite CW3 4 , 5 by mons the

effective setting given, $A_{p}=P_{c}$. The entire sequence of sub-paths taken by the current may be represented graphically, as shown below


Upon depressing the next key, CW5 advances one step to the next effective setting before completion of the enciphering circuit, and therefore, AL5 must be correspondingly advanced. But in which direction, to the left, or to the right? It will be seen that in the machine itself the movement of the wheel is opposite in direction to that followed by the letters on the periphery of the wheel. ${ }^{1}$ Hence, since NAL5 is the normal or straight alphabet, proceeding from left to "It will constantly be kept in mind that the assumption made regarding the cipher wheels is that they are in the "right-side-up" positio
right, to make the change in position of the sliding strip correspond with the change in position of the wheel, the strip must be advanced one space to the left, to the position indicated below:

| SET |  |
| :---: | :---: |
| BSXRZTKDNGCHMVOLYQEUPWJAIF LFS |  |
|  |  |
| ABCDEFGHI JKLMNOPQRSTUVWXYZABCDEFGH BDFCWPAOMSHXVIEUQZYGJTNLRKBDFCWPAO | AL1 |
| BDFCWPAOMSHXVIEUQZYGJTNLRKBDFCWP |  |
| WYDHKPÜQAJFOTCMIVZSEGLNRXBWYDHKP....) AL2 |  |
| ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLM.... HLSZVDJNXBTFMRPPAWOUGIQECKYHLSZVDJNXBTFM. ..\} AL3 |  |
|  |  |
|  |  |
| $\left.\begin{array}{l} \text { ABCDEFGHIJKLMNOPQRSTUVWXYZ. .. } \\ \text { DKHXWAFYORVBIMPTJEUQSCGLNZ... } \end{array}\right\} \text { AL4 }$ |  |
| ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLM.... $\}$ AL5FRISYADPLJUXZGKOBTWCVMHEQNFRISYADPLJUXZ |  |
|  |  |
| $\leftarrow \frac{\text { Direction }}{\text { of molion }}$ |  |
| TYOEUMXDFJQVKWBNSHCILRZAGP | RFS |
| SET |  |

Suppose the second letter to be enciphered is R. A condensed graphic representation of its encipherment is as follows:

In the foregoing graphic representation $L_{1}$ stands for the letter $L$ in the upper or normal component, and $L_{2}$ for the same letter in the lower or mixed component of Alphabet 1. The same designating characters also apply to the other letters in the diagram.

If the next letter to be enciphered were $\mathrm{E}_{\mathrm{p}}$, the graphic representation would be as follows:

$$
\underset{\mathrm{E}_{\mathrm{p}}}{\mathrm{LFS}} \Rightarrow \overbrace{\mathrm{~A}_{1} \rightarrow A_{2}}^{\mathrm{AL} 1} \Rightarrow \overbrace{\mathrm{E}_{1} \rightarrow \mathrm{E}_{2}}^{\text {AL2 }} \Rightarrow \overbrace{\mathrm{A}_{1} \rightarrow A_{2}}^{\text {AL3 }} \Rightarrow \overbrace{\mathrm{C}_{1} \rightarrow \mathrm{C}_{2}}^{\text {AL4 }} \Rightarrow \overbrace{J_{1} \rightarrow J_{2}}^{\text {AL5 }} \Rightarrow \mathrm{R}_{\mathrm{c}}
$$

This process is continued fon the succeeding letters in a similar manner, ALs being slid to the left one space for each letter. No shifting of CW $1,2,3$, or 4 will occur until N of RAW reaches the SET line, whereupon, as explained in paragraph 14, section II, with the next depression, CW1 as well as CW5 (also LAW and RAW) will be advanced one space. In this case since the first letter of the dispatch is enciphered with T of RAW at SET (Key: SIGNALS, effective setting $=$ SIGNAMT), then N will be reached with the encipherment of the 21st letter ( T to N , inclusive, in the normal alphabet equals 21 intervals). With the encipherment of the 22 d letter therefore, CW1 and CW5 will both be advanced one space each, and therefore AL1 and AL5 in the equivalent sliding strips must be advanced one space to the left to correspond with the displacement of CW1 and CW5.

The process of finding cipher equivalents is now the same as before. No second shifting of CW1 must be accounted for on the strips until the $22+26=48$ th letter of the dispatch
is to be enciphered, for once more (with the encipherment of the 47 th letter) N of RAW will have reached SET and CW1 will be advanced one space. This process of shifting AL1 to make it correspond with the successive displacements of CW1 is continued in like manner until the letter N of LAW reaches SET, which will be the $22+(26 \times 20)=542 \mathrm{~d}$ letter ( T to N, including T but not N, on LAW = 20 intervals). The 542 d letter will bring LAW to N and C I
to D . The 543 d letter will cause CW3 to advance one space, whereupon sliding AL3 must to D. The 543d letter will cause CW 3 to ad

It may be advisable to make an actual outline of the positions into which the successive letters fall, and the successive displacements to which the various wheels are subject during letters fall, and the successive displacements to which the various wheels are subject during
the course of the encipherment of a dispatch of say 600 letters, with the keyword SIGNALS. the course of the encipherment of a dispatch of say 600 letters, with the keyword It is recommended that the reader examine the following diagram very carefully. The numIt is recommended that the reader examine the following diagram very carefully. The num-
bers refer to the successive letters of the dispatch. At the top, the effective enciphering pers refer to the successive leters of the dispatch. At the top, the effective enciphering
positions of CW5 and RAW are given; at the left, those of LAW, CW1, and CW3 are given.

T $\begin{array}{ll}\text { U } & \text { K } \\ \text { V } & \text { L }\end{array}$

 $\begin{array}{ll}\mathrm{X} & \mathrm{N} \\ \mathrm{Y} & 0\end{array}$ e
18. Enciphering an example.-It is necessary that this chain of reasoning be noted very carefully, for in the subsequent analysis these factors must be considered in great detail. As practice in the procedure let the reader first encipher the following message by means of the sliding strips and the keyword MEANING, and then check it back by deciphering it:

## ammunition exhausted

19. Deciphering by means of sliding alphabets.-In decipherment by means of the sliding alphabets, the only difference in method is that now the analyst must proceed in the reverse direction in tracing paths, beginning with the cipher letter in RFS, progressing upward throug the five alphabets, and emerging at LFS. Each letter must first be located in the lower half or mixed component of each alphabet, carried to the upper halves or normal components, and so on Thus, to trace back the equivalents $A_{p}=P_{c}$ found with the initial setting SIGNALS, the steps are graphically illustrated as follows:


The correct encipherment is: ONZOV HLDGN XBVPW YMRN.

Section IV
analysis based ONLY UPON a KNOWLEDGE OF THE MECHANICS OF THE MACHINE

Introductory remarks Alphabets employ
demonstration Fundamental principle. $\qquad$

| 20 | Nature of table of basic cipher-text sequences.-. |
| :--- | :--- |
| 21 | The table of basic cipher-text sequences and the | 22 right fixed sequence.

20. Tntroductory reme when a
 me mechanics of the machine is available will be presented. Then, the method of analysi manner will appear subsequently
21. Alphabets employed in demonstration.-In demonstrating the principles of analysis use will be made of the alphabets given in paragraph 16, section II From the worki known alphabets, certain principles will be deduced From these deductions, inductive reaso ing will lead to the establishment of other principles, by means of which unknown alphabets may
 ing a complex system of cryptography.
22. Fundamental principle.-Consider the simplest case of all, viz, the successive encipherment of one letter 26 times, beginning with Z of CW5 at SET (so that A will be the effective setting on CW5) and with no displacement of the other cipher wheels occurring during the 26 successive encipherments. It follows from what has gone before that the electrical impulses originating in LFS will trace exactly the same path through CW1, 2, 3, and 4 every one of the 26 times, and will emerge from CW4 at one and only one of its RHCs every time, thus always entering CW5 from a fixed or constant point in BS5. The different cipher equivalents which will be produced for the 26 successive encipherments will therefore all be due sotely to the successive dis placements of CW5. For example, consider the successive encipherments of $A_{p}$ with the initial effective setting of the wheels given as OAAAAAO. Tracing the encipherments, or operating the keyboard of the machine, the first equivalent of $A_{p}$ will be $Y_{c}$; the second one, $O_{c}$; the third one $\mathrm{N}_{\mathrm{c}}$, and so on, yielding the following sequence:

If the encipherment has been accomplished by means of the sliding strips, it will be observed that in every one of the foregoing 26 encipherments the current emerges from E of MAL4, and that in every one of the foregoing 26 encipherments the current emerges from E of MAL4, and contact of BS5. Now note the following very carefully, for it involves the essence of this whol analys
(1) No matter what the rotatory permutation of CW1,2,3, and 4 may be;
(2) No matter what plain-text letter is being enciphered 26 consecutive times, designate by the symbol $\theta_{\mathrm{D}}$;
(3) Providing no displacement of CW1, 2, 3, or 4 occurs during the 26 successive encipherments;
(4) If the current enters CW5 from the eighteenth contact of BS5 and
(5) If A of NAL5 is at the SET line, the series of cipher equivalents for the 26 consecutive encipherments of $\theta_{\mathrm{p}}$ will be the sequence
Y ONDSWMAUZXFLQKGXVHRBTECJP
To repeat, it makes absolutely no difference what the rotatory permutation of the first four cipher wheels may be, if the other conditions set forth above hold true, the 26 successive cipher equivalents of $\theta_{\mathrm{p}}$ will coincide with the sequence YOND. . . The latter may be regarded as absolutely fixed for the given CW5. For example, set up the following permutation of the wheels: KTHWKZN, which may be regarded as being a random one; depress the universal bar so as to advance CW5 to A, and CW1 to the next position so as to provide for a complete sequence of 26 encipherments; hold the universal bar down with the left hand; and then with a finger of the right hand find that key which will cause Y to be illuminated on the lightboard. It will be found that $F_{p}$ will produce 1 . Now release the universal bar and depress $F$ successively 25 more times. The sequence of equivalents will be YOND.

Both of the procedures in establishing the preceding sequence started with CW5 at an initial point, which for convenience was selected as A. But if the sequence is started with some setting other than A, the only difference is that the initial letter of the sequence is no longer $Y$, but some other letter of the same sequence: invariably, there will be produced this same YOND... sequence. For example, with the setting QWKANFN, depressing $K_{D}$ successively yields the following sequence:

which, it will be noted, is exactly the same as the YOND. . . sequence given above, but with a different initial point. In other words, the YOND... sequence may be regarded as being in the nature of a cycle, which can be initiated at any point. For convenience, therefore, this cycle of letters, no matter what its initial point is, will be called the YOND
sequence.

There are $26^{4}$ or 456,976 rotatory permutations of CW1, 2, 3, and 4 . For every single one of them there will be a certain key of the keyboard, depression of which will produce the YOND. sequence because the current which results from depressing that particular key will enter the LHC's of CW5 from the eighteenth fixed contact of BS5. What that key will be for each one of hose 456,976 permutations of CW1, 2, 3, and 4 is of no importance at this point of the analysis.

Now it is obvious that BS5 has 26 and only 26 fixed contacts through which current can emerge from a RHC of CW4 and enter into the LHC's of CW 5. It follows, therefore, that there can be 26 and only 26 such final cipher sequences for any setting of CW1, 2, 3, and 4; and thus, for any one of the 20 letters of the alphabet, no matter what the initial positions of the first four cipher wheels may be, the consecutive depression of any one key, for 26 times, with no intervening displacement of CW1, 2, 3, or 4, will yield a sequence of cipher equivalents that is absolutely fixed. The
complete set of sequences, designated hereafter as the BASIC CIPHER-TEXT SEQUENCES, ${ }^{1}$ is given in the accompanying table 1
${ }^{1}$ The table of basic cipher-text sequences will be entirely different when the machine is set with the indicator at REVERSE.
table 1.-TABLE OF basic CIPHER-TEXT SEQUENCES ABCDEFGHIJKLMNOPQRSTUVWXYZ 荮 Y O NDSWMAUZXFLQKGXVHRBTECJPXI QISFTDPJVAWNYJBLGHEXRKOUMC J Z UVEWNYHIMWCGORDQPSXFJZBALK WT A PVIGUFYWSMZKBNOLETCQHJIRDIX C Q O J YVWPNHEVSZTIMFABUXDLKR VG RGWULZAQOVFTCPHYENMDIYXKSJYB DAPTWMBVEROLUXCFMQZUNTKGYS UH K U B S O C L X B I P E ZFVTHDJQANGRWM BY LH JYFTQKGBSOQNRPCUDZWEMXIV QA S C U L Z F S R Y M G Q W O I J V K P H T A N D B X S E
 OFKMHL JTSCIXGDUZBRWAPVYENA A Q H J N WEQOKBTSCUKHAYLXJGNMDFRWZTKP GTYBXNKUZERMDIJVAMGSLWPOHFMC MNHEIRDNLTUAJKYCFQVGSPZBXWNO ESFHBXGMADJRVWPDKCZNYUIQTODL IMRAJHZOXPVBELQKWTCYGSFNDH HU BDIZVOCLRFTJXGSANPYWEMHPUQ PK NXZGPKUWQOLYIEMHDJREFBCVATES JWXCRQYHILDPFMANZBGTKOUSGEGV XZGQCAEDTKNURVWBYIOPHJSFCLCM TK L PM J O B H XA W N S ER U Y I V C Q L Z F G
23. Nature of table of basic cipher-text sequences.-It will now be shown that the table of basic cipher-text sequences given in the accompanying table 1 may be regarded as being merely a set of 26 secondary alphabets resulting from the sliding of two primary alphabets against each other. One of the primary alphabets (only when indicator is at DIRECT) is the right fixed sequence, RFS, the other is MAL5 (which merely represents the series of RHC's of CW5). There is, however, a slight difference between the mechanics of the system of producing the secondary alphabets in this case, and the mechanics of the usual or ordinary systems of producing secondary alphabets. In the usual systems each secondary alphabet is produced by two primary alphabets. In this system, not only are the initial settings of the two primary alphabets differt for alphabets is regularly displaced ine inval in deriving each successive letter of each secondary alphabet. To demorte set MAL5 against RFS so that their initiol letters are opposite each other, thus:

MAL5_--- FRISYADPLJUXZGKOBTWCVMHEQNFRISYADPL.. RFS...--- TYOEUMXDFJQVKWBNSHCILRZAGP

Now construct an enciphering alphabet by successively displacing the upper component MAL5, one interval to the left and writing down the equivalents found on RFS for the letters A, B, C, ... of MAL5. For this initial setting the sequence is the following:

M NHEIRDNLTUAJKYCFQVGSPZBXW
Comparing this with the eighteenth sequence in table 1, it is seen to coincide with it.
If the two primary sequences are set at the following initial positions, and the same process of finding cipher equivalents is followed, the sequence obtained coincides with the eighth sequence of table 1 .
MAL5_-.-FRISYADPLJUXZGKOBTWCVMHEQNFRISYADPL.. RFS.....- TYOEUMXDFJQVKWBNSHCILRZAGP
Secondary alphabet:
UBS OCLXBIPEZFVTHDJQANGRWMK
The reason why this secondary alphabet begins with the second letter of the corresponding basic cipher-text sequence of table 1 is, of course, that the upper primary alphabet, MAL5, was at a setting equivalent to the second position of CW5, the setting when A of its normal alphabet component is at SET being considered the first position.

It is therefore apparent that (1) the table of basic cipher-text sequences is only a table of secondary alphabets produced by the sliding of two mixed alphabets against each other, and (2) this being the case, the secondary alphabets are all interrelated in some manner which ought to permit of the reconstruction of all of them having given only two of them
24. The table of basic cipher-text sequences and the right fixed sequence.-As a matter of fact, a clear way of looking at the cipher mechanics of the machine in encipherment (DIKEC") is to consider that all that the machine does is to apply a sort of and the right fixed sequence acording to an absolutely definite intervallength (when the wiring i unchanged) . To demonstrate exactly what is meant, take the first sequence of table is YOND. . . . refer to RFS and count the number of intervals between the successive letters of the YOND. . . . sequence as they are located on the RFS, always counting from left to right. Thus, YOND... sequence as they are located on the RFS, always counting from left to right. Thus, intervals, and so on. The following sequence of intervals results:

RFS...- TYOEUMXDFJQVKWBNSHCILRZAGPT

This sequence of interval-numerals is the measuring rule. Apply it to RFS at any point and see the distribution of letters it yields. For example, start with A of RFS. The firs interval length is 1 ; hence the letter which occupies the first position to the right of A, viz, G, is the second letter. The next interval length is 13 ; hence the letter which occupies the thirteenth position to the right of G in RFS is the third letter, and so on. The following sequence results
A GVEKJYITCOUSXFLODWHQZPBMR

Compare this with the eleventh sequence of table 1 and it will be seen to coincide with it, the initial point merely being different.
${ }^{1}$ See sec. XI of Tr. Pamphlet No. 3, and Riverbank Publications Nos. 15 and 21 , listed in the bibliography
to Tr. Pamphlet No. 3. to Tr. Pamphlet No. 3.

It is seen, therefore, that the measuring rule merely lays off a sequence of specific distances on RFS, and this sequence of distances is always constant. Now then, what is the measuring rule after all? It is merely the equivalent of CW5. To show that it is, consider the basic sequence YOND ... The first letter $Y$ is given by the setting as follows:

The letter R of MAL5 is the one involved. Now slide AL5 one space to the left; 0 is the second letter of the basic sequence, and the letter of MAL5 that is involved is S. Slide AL5 once more to the left; N is the third letter of the basic sequence, and the letter of MAL5 that is involved is T. Those letters of MAL5 which are successively involved are seen to follow the sequence of the normal alphabet, A, B, C, ... Z. Now count the number of intervals between A, B, C, ... as they appear on MAL5, always counting to the right. The interval re as foilows:

But since CW5 moves one space to the left each time a letter is enciphered, and since the determination of the intervals was made by counting always to the right, a deduction of one interval should be made from each of the intervals above. This yields the following:

11-3-13-17-3-13-9-6-7-5-20-13-4-16-18-17-3-2-14-19-10-24-19-19-8-19
$10-2-12-16-2-12-8-5-6-4-19-12-3-15-17-16-2-1-13-18-9-23-18-18-7-18$
Compare this with the original sequence of interval-lengths given above (p.20), and it will be seen to be the same, merely displaced nine spaces to the left.

The question is asked, why is it that those letters of MAL5 which are successively involved in yielding a basic sequence follow one another according to the normal alphabetic sequence? The answer is quite clear: since the current always enters CWo from the same fixed contact pested

It is apparent therefore that a very clear way of looking at the whole problem is this. RFS
m endless or cyclic series of letters, and CW5 is likewise an endless or cyclic mensuring rule with a definite set of graduations marked upon it Applying the two circles to each other set of 26 interval expressions of the letters of RFS are yielded by the measuring circle, CW5 The only part played by CW1, 2 s and 4 is that concerned with the initial point at which the measuring rule is applied to RFS. the resultant of the interaction of CW1, 2, 3, and 4 determines merely the initial point of application of CW5 to RFS

It also follows that if CW5 is replaced by another wheel, the "graduations" on the new measuring rule are no longer the same as before, and different interval expressions of the same RFS will be obtained. Therefore the sequences of cipher resultants will be altogether different from those of before, and a new table of basic cipher-text sequences will result. Now in a given machine there are five cipher wheels, and the wheels are interchangeable. With five. wheels, as stated before, 120 different horizontal permutation arrangements on the shaft are possible (the
wheels all right-side up), but the fifth wheel in any of these 120 arrangements can be only one of five wheels. Now it was demonstrated in paragraph 22 that the point at which the electric current enters the LHC's of CW5 from BS5 is the only determining factor in establishing the particular sequence of cipher equivalents resulting from the successive depressions of any key. No matter, therefore, what the rotatory permutation of CW1,2,3, and 4 is, providing no displacert of will therefore that it makes no difference whatever as far as this particular factor is concerned, what the horizotal permutation first four cipher wheels is, whether they are in the order $1-2-3-4$, or $4-2-1-3$, or any other permutation, the electric current will always enter the LHC's 1-2-3-4, 5 of CW5 from one and only one contact of BS5, providing no displacement of the first four wheels occurs during the successive 26 encipherments of a particular letter.

This question raises itself: if the final cipher resultants are determined merely by the interaction of CW5 and RFS, what then is the function of the other cipher wheels?

The answer is that the particular sequence or succession in which the basic sequences follow each other in the table is determined by the particular combination of rotatory and horizontal permutations of CW1, 2, 3, and 4 upon the shaft. For the arrangement studied, viz, where CW1 occupies the first position on the left, CW2, the second, and so on, and where the cipher wheels are set to A all the way across, the first basic sequence corresponding to the encipherment of $A_{D}$, is YOND...; the second basic sequence, corresponding to the encipherment of $B_{D}$ is QISF... , and so on. For a different rotatory permutation of CW $1,2,3$, and 4 upon the shaft, or for the same rotatory permutation but with a different horizontal permutation of CW $1,2,3$, and 4 , the order in which the 26 basic cipher-text sequences corresponding to the encipherments of $A, B, C \ldots Z$ will fall will be entirely different, but the sequences themselves will be exactly the same as before. In short, the only function of CW1, 2,3 , and 4 is that concerned with determining the particular order in which the individual sequences of the whole table of basic cipher-text sequences are produced.

It has been stated above that each different cipher wheel, when employed in the fifth position, will yield a different table of basic cipher-text sequences, even though RFS remains the same. If there are $n$ cipher wheels available, then there can be $n$, and only $n$ different tables, since the horizontal and rotatory permutations of the other four cipher wheels act as CW $1,2,3$, and 4 and have nothing to do with the sequences of the tables.

Assuming, however, the encipherment of a series of dispatches all prepared by means of the same horizontal permutations of the cipher wheels, it follows that one and only one table of structed if two and only two of its sequences are known, or if only one sequence and the RFS are known, and that the entire table is in reality composed of but one sequence of 26 letters, which when reconstructed destroys the entire secrecy of a system involving almost twelve million when reconstructed destroys the entire secrecy of a system involving almost twelve minion
secondary cipher alphabets. One might even go further and say that under certain circumstances the entire secrecy of a system embracing over 91 billion secondary alphabets is dependent upon maintaining secrecy with respect to a single sequence of but 26 letters.
${ }^{1}$ If it is taken into consideration that each wheel may be inserted in an "upside-down" position when acting as CW5 then there are $2 n$ different tables possible.

Section V

## THE TABLE OF BASIC CIPHER-TEXT SEQUENCES

Effects of repetitions of plain-text letter | Par. |
| :---: |
| -25 |
| 25 | Summary of preceding analysis Par.

$--\quad 27$
25. Effects of repetition of plain-text letters.-Assume for the moment a dispatch of 26 letters consisting exclusively of the letter $A_{p}$, enciphered by the effective setting OAAAAAO of the cipher wheels. The cipher text will be YOND..., one of the basic cipher-text sequences. Now assume a dispatch of 26 letters consisting exclusively of the letter $B_{p}$, also enciphered by the same effective setting. The cipher text will be QISF..., another one of the basic sequences Now assume a dispatch of 26 letters consisting exclusively of the letters A and B, alternately, also enciphered by the effective setting OAAAAAO. The cipher text will be as follows:
$\begin{array}{llllllllllllllllllllllllll}\text { Plain_.... } & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A \\ C & B \\ \text { Cipher... } & Y & I & N & F & S & D & M & J & U & A & X & N & L & J & K & L & X & H & H & X & B & K & E & U & J \\ C\end{array}$
This consists merely of alternate letters of the two basic sequences YOND... and QISF..., as is shown herewith:
$\begin{array}{lllllllllllllllllllllllllll}\text { Plain...... } & \text { A } & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B & A & B \\ \text { Cipher_-. } & \text { I } & I & N & F & S & D & M & J & U & A & X & N & L & J & K & L & X & H & H & X & B & K & E & U & J & C\end{array}$

Now assume a dispatch of 26 letters, consisting exclusively of the letters $T$ and $D$ alternately, enciphered by the initial effective setting OVGMBAO. It is as follows:

## 

The cipher text is exactly the same as for the alternate $A_{p}-B_{p}$ dispatch, with an entirely different setting or key. The plain-text letters are different in the two dispatches but the cipher etters are identical. In fact, an identical cipher sequence can be obtained for any pair of letters are identical. In fact, an identical cipher sequence can be obtained for any pair of
different letters whatsoever when the proper initial setting is chosen. Why is this? Is it not different letters whatsoever when the proper initial setting is chosen. Why is this? Is it not
due to the effects that mere repetition of plain-text letters produces in this machine, regardless due to the effects that mere repetition of plain-text letters produces in this machine, regardless
of what letters are involved? What has gone before should make this clear. In other words, of what letters are involved? What has gone before should make this clear. In other words,
what concerns the cryptanalyst in this case so far, is not the question as to what letter is repeated in the plain text but whether or not a letter, any letter, is or is not repeated. The results of repetition of any letter are predetermined, and the cipher equivalents occupy definite positions in one of the basic cipher-text sequences. The identity of the repetition is of no immediate concern, but the existence or nonexistence of the repetition is of the highest importance.

Now consider what happens in the encipherment of intelligible text. Let the dispatch be THE ELEMENTS OF THE SCIENCE OF CRYPTANALYSIS..., enciphered with the initial effective setting OAAAAAO. The results are as follows for the first 26 letters:

Plain_.... THEELEMENTSOFTHESCIENCEOFC Cipher...- I U O J UV JPFPJSCLVIKSDBMZDJSK

From what has gone before, it is to be expected that-
(1) The cipher equivalents of identical letters will all belong to the same basic cipher-text sequence, and will fall into definite positions in that sequence
(2) There will be present elements of as many basic cipher-text sequences as there are different plain-text letters; and
(3) As a corollary of the foregoing, the cipher equivalents of dissimilar letters will never exhibit coincidences with the same individual basic sequence.
With these three principles in mind, examine the foregoing encipherment
Take the basic sequence which begins with I (the 20th in table 1) and apply it to the cipher text. Coincidence of the 1 st , 10 th, and 14 th cipher letters with the 1 st, 10 th, and 14 th letters of the basic sequence is noted as stated in (1) above, and noncoincidence with all the other letters also is noted, as stated in (3) above.

Here the coincidences are due to repetitions of one letter, $T_{D}$; the noncoincidences represent letters which can not be $\mathrm{T}_{\mathrm{p}}$. If one did not know the plain text, but had merely the two bottom lines, one could nevertheless state definitely that the 1st, 10 th, and 14th letters of the plain text are identical, and that whatever this plain-text letter is, it does not appear elsewhere in that line. The frequency of that plain-text letter is thus directly indicated by the number of coincidences with the basic sequence.

Now take the basic sequence which has U for its second letter (it is the 8th of table 1), and apply it to the cipher text. The coincidences are underlined below

Plain_-
THEELEMENTSOFTHESCIENCEOFC
Cipher
I
$K$
$\underline{U}$
B S S O C L

$$
K \underline{U} B S O C L X B I P E Z F \underline{V} T H D J Q A N G R W M
$$

Now take the basic sequence which has 0 as its third letter (the fifth of table 1), and apply it to the cipher text. The coincidences are underlined below:

Plain_-.......- THEELEMENTSOFTHESCIENCEOFC
Cipher-
IU O V JPFP JSCOC
Cipher C Q $\underline{O} \underline{J} Y \underline{V} W \underline{P}$ N HEVS Z T I M FA B U X

This process can be continued until all coincidences have been noted. For the purpose of graphic representation, all the coincidences have been reassembled into the one diagram below, those belonging to the same basic sequence being indicated by identical numbers.

Plain--------- THEELEMENTSOFTHESCIENCEOFC


This procedure shows, therefore, that intelligible text when enciphered by the machine produces cipher text whose letters when arranged in lines of 26 can be distributed into definite positions in as many basic cipher-text sequences as there are different plain-text letters enciphered in each line; that those letters of the cryptogram which coincide with the letters of the same basic cipher-text sequence represent encipherments of identical plain-text letters; and that the frequency of tained by present.
26. Use of the table of basic cipher-text sequences.-Let us assume for the moment that the table of basic cipher-text sequences applying to a series of dispatches has been obtained in some illegitimate manner by capture, or otherwise. It is obvious that one could immediately determine those letters in each line which represent repeated letters of the plain text. For example, in the case of the enciphered message on page 24, one would be able to underline the example, in the case of the enciphered message on page 24 , one would
repetitions in exactly the same manner as was done there. One would know then that the 1st, repetitions in exactly the same manner as was done there. One would know then that the 1st,
10th, and 14 th plain-text letters were the same; the 2 and 15 th, and so on. The analyst could do this for all the lines of cipher text. The result would be that the cipher text would have been decomposed into a series of single-alphabet substitution ciphers, the solution of which would not be very difficult, as will subsequently be fully illustrated. Thus, it is apparent that the entire secrecy of the machine can be almost entirely destroyed if the table of basic cipher-text sequences is known to, or can be reconstructed by the enemy.
27. Summary of preceding analysis.-The most important facts and conclusions that were developed in the foregoing analysis may be conveniently summarized as follows.
(1) Cipher text produced by the machine is composed of the elements of 26 and only 26 basic cipher-text sequences.
(2) Every line of 26 letters of cipher text is composed of the spatial elements of as many different basic cipher-text sequences as there are different letters in the line.
(3) All that the machine does is to determine (in what appears to be a random, haphazard manner) the particular basic sequences that will be represented in each line of cipher text.
(4) The fifth cipher wheel in interaction with RFS produces the 26 basic sequences; the other four cipher wheels merely determine the permutations of the horizontal lines of the table of basic cipher-text sequences, or in other words, the order in which the basic cipher-text sequences follow each other in the encipherment of a dispatch.
(5) Possession or reconstruction of the table of basic cipher-text sequences will enable the cryptanalyst to distribute the letters of the text of cryptograms into a series of single-mixed substitution alphabets, which can be solved rather readily

Section VI

## MATHEMATICAL THEORY OF ANALYSIS

Application of the general principles of frequency ${ }^{\text {Par }}$
Par.
28 Development of
to the problen to the problem.
28. Application of the general principles of frequency to the problem.-In an actual tabulation of 100,000 letters occurring in telegrams of an administrative nature handled by the War Department Message Center on one day, the following distribution was found:

TAble 2

| E 12,604 | 0 | 7,408 | C | 3, 345 | M | 2, 534 | B | 1,146 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T 9, 042 | A | 7, 189 | H | 3, 287 | Y | 2, 099 | X | 469 |
| R 8, 256 | S | 5,759 | F | 2, 994 | G | 1,795 | K | 353 |
| I 7,572 | D | 4, 029 | U | 2, 993 | W | 1,401 | Q | 318 |
| N 7,558 | L | 3, 549 | P | 2,661 | V | 1,340 | J | 198 |

This frequency table, based as it is upon a fairly large number of letters, may be considered representative of the normal or typical constitution of telegraphic English text.
What this normal frequency table means, of course, is this: If a volume of telegraphic English text totalling 100,000 letters is examined, there will be found approximately $12,600 \mathrm{E}$ 's, $9,000 \mathrm{~T}$ 's, $8,300 \mathrm{R}$ 's, and so on. In other words, the data based upon 100,000 cases may be considered as giving a true picture of the constitution of any large volume of such text, and in any equivalent volume of text similar in nature, practically the same reative proportions of occurrences will be found to exist as were found in the 100,000 cases examined. Now if the 100,000 letters of which the text is composed were placed in a hat, and thoroughly mixed, the chances of
drawing an E by a random selection are $\frac{12,604}{100,000}$ or approximately 0.126 ; that is, according to the laws of probability in 1,000 successive drawings, a total of 126 E's would be chosen. Mathematically stated, $P$ (the probability) for selecting an E is 0.126 . Likewise, $P$ for $\mathrm{T}_{\mathrm{D}}$ is 0.090 , $P$ for $R_{p}$ is 0.083 , and so on. Now, if instead of placing the letters in a hat, one should have $P$ for $R_{p}$ is 0.083 , and so on. Now, if instead or paciog should close one's eyes and at random point a pencil at one letter in the lext just as it stands, it is obvious that $P$ for E would etill be somewhere in the neighborhood of 0.126 ; for $T_{D}$ it would still be approximately 0.090 ; and so on, because, considered in its broadest aspects, the text is composed of a great variety of words, and so far as words are concerned, they are made up of such a diversity of permutations and combinations of letters that plain text can almost be regarded as being a random assortment of letters in the relative proportions given above. ${ }^{1}$

Now proceed one step further. Suppose two pencil points be directed, at random, with one's eyes closed, at intelligible text; what are the chances for designating two E's simultaneously, or two T's, or two of any other letter? According to the mathematical theory of probability, the probability that both of two independent events will occur together is the product of thei ${ }^{1}$ This, of course, is not strictly true, and is discussed in section IX, p. 51.
separate probabilities. Hence, the chances for simultaneously designating two E's are $0.126 \times$ 0.126 , or 0.016 ; the chances in the case of two T's are $0.090 \times 0.090$, or 0.008 , and so on.

Again, according to the theory of probability, the probability of the occurrence of several events which cannot occur together is the sum of the probabilities of their separate or individual occurrences. Thus, if $p, q, \ldots$ denote the separate probabilities of different events, the probabiity, $r$, that one of the events whin happen is $P=p+q+$ Since the probability fo random selection or two E is 0.010 , hat for two Ts is 0.00 , and so on, the probabiity for the ramb proo 's That is to say the probability of directing two pencil points simultanaly two 2 s. Two identical letters) is the sum of the separate probabilities for repetition of any letter of the alphabet. In table 3 the separate probabiiitics and their total are shown. The total of the separate probabilities for repetition, 0.066 , means that in 1,000 cases where a pair of letters is selected or designated at random there will be 66 cases in which both members of the pair so selected will be the same letter; in other words, $P$ for the simple occurrence of repetition, (hereafter denoted by the symbol ${ }_{r} P$ ) regardless of what the repetition maybe, is 0.066 . As a matter of fact, the 66 cases will be composed of 16 cases where the repetition is the letter $E, 8$ cases where the repetition is $T, 7$ where it is $R$, and so on, in accordance with the last column of table 3 . This, however, does not concern us at present.
table 3

| Letter | Frequency | $P$ for separate <br> occurrence | $P$ for reneti- <br> tion (P2) |
| :---: | ---: | :---: | :---: |
| E | 12,604 | 0.126 | 0.016 |
| T | 9,042 | .090 | .008 |
| R | 8,256 | .083 | .007 |
| I | 7,572 | .076 | .006 |
| N | 7,558 | .076 | .006 |
| O | 7,408 | .074 | .006 |
| A | 7,189 | .072 | .005 |
| S | 5,759 | .058 | .003 |
| D | 4,029 | .040 | .002 |
| L | 3,549 | .035 | .001 |
| C | 3,345 | .033 | .001 |
| H | 3,287 | .033 | .001 |
| F | 2,994 | .030 | .001 |
| U | 2,993 | .030 | .001 |
| P | 2,661 | .027 | .001 |
| M | 2,534 | .025 | .001 |
| Y | 2,099 | .021 | .000 |
| G | 1,795 | .018 | .000 |
| W | 1,401 | .014 | .000 |
| S | 1,340 | .013 | .000 |
| X | 1,146 | .011 | .000 |
| K | 469 | .005 | .000 |
| Q | 353 | .004 | .000 |
| J | 318 | .003 | .000 |
| Z | 198 | .002 | .000 |
| Total_......- | 100,000 | 101 | .001 |

On the other hand, since the probability that an event will not happen is the difference between unity and the probability that it will happen, or ( $1-P$ ), the probability, $P$, for nonbetween unity and the probability that it will happen, or $(1-P)$, the probability, $P$, for non-
repeafter denoted by the symbol $n_{r} P$ ) is $1-0.066$, or 0.934 . That is, in 934 of the 1,000 cases, the two letters selected at random will be different.
29. Development of mathematical theory applicable to the problem.-The relation of this mathematical theory to the problem in hand will now be studied.

The normal frequency table shown in table 2 may be considered as being heterogeneous in this sense: It is composed of letters of widely different but rather definite or constant frequencies which gives the table a characteristically irregular "crest and trough" appearance. It is this very heterogeneity or irregularity of the table which leads directly to the solution of the simplest type of substitution cipher, that known as the single, mixed alphabet type. Any really scientific cryptographic method has for its aim the suppression of the frequency characteristics of the the better the cryptographic method, the more complete is the suppression. In a perfect cipher system, the frequency table for the cryptographic text should be completely homogeneous or regular in this sense: (1) That all the letters of the alphabet should be represented, (2) they should occur with practically equal frequencies, thus suppressing the appearance of any "crests and troughs", and (3) there should be no easy way of decomposing the homogeneous or regular table for the cryptographic text into one or more heterogeneous or irregular tables such as apply to singlealphabet substitution ciphers. The cipher machine under consideration has been designed to produce such a result as nearly as possible, and it really does so to a very high degree. The following very homogeneous single-frequency table is based upon ten cryptograms produced by the machine, no two messages being in the same rotatory key

Bearing in mind the three characteristics stated above, with respect to the frequency table for the text of a cryptogram produced by a theoretically perfect cipher system, cxamine the foregoing table. It will be noted (1) that all the letters of the alphabet are represented, and (2) that they occur with practically equal frequencies. There remains to be considered, therefore, only the third factor above-mentioned. Can this frequency table be decomposed into a multiplicity of tables exhibiting the heterogeneity and irregularity of normal single-a!phabet distributions? That is the problem before the analyst in this case.

Now in any letter-for-letter substitution in a cryptographic system resulting in the production of homogencous text in the sense stated above, the values ${ }_{r} P=0.066$ (as explained in par. 28) and ${ }_{n r} P=0.934$, must still hold true. This must be so because each cipher letter represents but one plain-text letter at a time, even though the plain-text letter may in each separate case be any one of the 26 letters of the alphabet. Taking the cipher text as it stands, according to the theory of probability as applied above, in 66 cases out of every thousand, any two cipher letters selected at random will represent the same plain-text letter, and in 934 cases out of every thousand, any two letters selected at random will not represent the same plain-text letter. analysis what the repeated plain-text letter is, because it was demonstrated in sections IV and $V$,

 this basic sequence, no matter what the plain-text letter be, its repetition in the next succeeding
$\qquad$
position will produce $0_{c}$; in the next, $N_{c}$; in the next $D_{c}$ and so on. The mathematical theory developed above states that 66 cases out of 1,000 will be repetitions of letters, and these may be repetitions of 2 As 2 BS 2 C setc. Now perfectly permissible to assume 1,000 cases where her graphic text produced by the maline so that 26000 lines graphic ene machine, and properly arranged, about 1,000 of these lines will have $A_{c}$ as their initial letter, 1,000 lines beginning with $Y$ Since the mathematical theory postulates that 66 out of 1,000 cases of a pair of letters selected at random will be repetitions, it follows that if a tabulation is made of only those letters which immediately follow $\mathrm{Y}_{\mathrm{c}}$ in the 1,000 lines beginning with that letter, then in 66 cases the letter $0_{c}$ should appear, and in 934 cases the letter should be some letter other than $0_{c}$. The reason that $0_{c}$ will be the letter representing a repetition of the first plain-text letter in the line is, of course, that in this particular basic sequence 0 immediately succeeds $Y$ in the YOND. . . sequence. Likewise, if a tabulation is made of only those letters which are in the second position after $\mathrm{Y}_{\mathrm{c}}$ in those 1,000 lines, then in 66 cases the letter $\mathrm{N}_{\mathrm{c}}$ should appear, and in 934 cases the letter should not be $N_{c}$. The same reasoning applies to tabulations made for any position after $Y_{c}$, and the letter which should appear 66 times out of 1,000 should be the letter which actually occupies that position in that particular basic sequence. The production of each letter in each of these cases is absolutely determined by the mechanicoelectrical features (including the wiring, of course) of the machine, and by the mechanics of the English language: for the first position after Y the letter representing a repetition must be 0 ; for the second, $N$; for the third, $D$; for the fourth, $S$; and so on, yielding sequences

As before, for convenience, the symbol $\theta_{c}$ will again be used as a symbol to designate any unspecified letter of the cipher text. The symbol ${ }_{r} \theta_{\mathrm{c}}$ will be used to designate the cipher equivalent of a repetition; for example, in the case of the YOND. . . sequence ${ }_{r} \theta_{c}$ for the first position after $Y_{c}$ is 0 ; or briefly stated $\theta_{c}$ in $Y_{c} r \theta_{c}$ is $0 ; \theta_{c}$ in $Y_{c} \cdot r_{\mathrm{c}}$ is $N ; \theta_{c}$ in $Y_{c} \ldots \theta_{c}$ is $D$; and so on. The symbol ${ }_{\mathrm{n}} \theta_{\mathrm{c}}$ will be used to designate the cipher equivalent of a nonrepetition

Now let attention be concentrated upon $\theta_{\mathrm{c}}$ for a given position in the YOND. . . sequence, for example, $\theta_{c}$ in $\mathrm{Y}_{\mathrm{c}} \ldots{ }_{\mathrm{r}} \theta_{\mathrm{c}}$. In 66 cases out of 1,000 (the ${ }_{\mathrm{r}} \theta_{\mathrm{c}}$ cases) $\theta_{\mathrm{c}}$ will be D ; in 934 cases (the ${ }_{\mathrm{nr}}{ }^{4} \theta_{\mathrm{c}}$ cases) it will not be D . Of the $66 \mathrm{Y}_{\mathrm{c}}^{1} \ldots{ }_{\mathrm{r}}{ }_{\mathrm{c}} \theta_{\mathrm{c}}$ cases, a certain portion of them will represent the occurrence of $E_{D} . . E_{p}$, another portion will represent the occurrence of $T_{D}{ }^{2} .{ }_{p}{ }^{234}$, another of ${ }_{R_{p}}^{1} .2 R_{p}$, and so on, each in proportion to its specific probability of repetition. But of the 934 $\mathrm{Y}_{\mathrm{c}} \ldots \mathrm{nr} \theta_{\mathrm{c}}$ cases, in each case the occurrence of two different letters is involved. A certain portion of them will represent the occurrence of $E_{p} \ldots . T_{D}$; another will represent the occurrence of $T_{D}{ }^{1}{ }^{2324} . E_{p}$; ${ }^{1}$ The fact that each of these repetitions occurs with a characterist ic frequency is innuaterial in this comnecelements.
 probability of occurrence. Now in each of these $934 Y_{4}^{1} Y_{c}^{23}{ }_{4}{ }^{4} \theta_{c} \theta_{c}$, or nonrepetition cases, $\theta_{c}^{4}$ may be any letter except $Y$ and $D$; what $\theta_{c}$ will be in each case is determined solely by the particular setting of the cipher wheels and by the plain-text letter which $\hat{\theta}_{\mathrm{c}}$ represents. For example, consider only the plain-text nonrepetition case ${ }_{1} \mathrm{E}_{\mathrm{D}} \ldots \mathrm{T}_{\mathrm{p}}$, whose probability of occurrence is $0.126 \times 0.09$, or 0.011 . Among the $934 \mathrm{Y}_{\mathrm{c}}{ }_{1}{ }^{2}{ }_{n \mathrm{nr}} \theta_{\mathrm{c}}$ cases there will be eleven cases in which
 these 11 cases exactly what $\theta_{\mathrm{c}}$ will be is determined solely by the particular rotatory permutation
 another $Y_{c}{ }_{c} . C_{c}$, and so on. The same will be true with respect to any other $Y_{c}{ }_{n n}{ }_{n \pi} \theta_{c}$ case such as $E_{p}^{1} \ldots R_{p}, T_{p}, \ldots, E_{p}$, and so on. In other words, the second member of the two cipher equivalents in the $934 \mathrm{Y}_{\mathrm{c}} \ldots{ }_{\mathrm{n}} \theta_{\mathrm{c}}$ cases can be any one of the 26 letters of the alphabet except Y and D . (An exception will be discussed in par. 83 , sec. XV.) That is, in the 934 cases of $\mathrm{Y}_{\mathrm{c}}{ }^{1}{ }^{23}{ }_{\mathrm{nr}}{ }_{\mathrm{n}} \theta_{\mathrm{c}}$, the ${ }_{\mathrm{ur}} \theta_{\mathrm{c}}$ 's or cipher letters which are the second members of the pairs representing cases of nonrepetitions, are distributed over 24 letters of the alphabet, and assuming a perfect homogeneity of text, ${ }_{\theta_{c}}$ in those 934 cases will be every one of the 24 letters an equal number of times, viz., $934 \div 24$, or approximately 39 times. But in the same 1,000 cases of $Y_{c}{ }^{1}{ }_{r}{ }^{2} \theta_{c}$, the $66{ }_{r}{ }_{\mathrm{r}} \theta_{\mathrm{c}}$ cases, or cipher letters which are the second members of the pairs representing cases of repetitions, are not distributed throughout the alphabet, but always yield the same cipher letter, which in this case is $D_{c}$. That is, the frequency of $\theta_{\mathrm{c}}$ in $\mathrm{Y}_{\mathrm{c}} \ldots{ }_{\mathrm{r}} \boldsymbol{\theta}_{\mathrm{c}}$ is 66 , whereas the frequency of ${ }^{4} \theta_{\mathrm{c}}$ in $\mathrm{Y}_{\mathrm{c}} \ldots{ }_{\mathrm{n}}{ }^{2} \theta_{\mathrm{c}}$ is only 39 , practically only two-thirds as large. It follows, therefore, that D should stand out prominently among all the other letters when a tabulation of 1,000 cases of $Y_{c}^{1} \ldots \theta_{c}{ }^{234}$ occurrences is made. Or, stated in other words, the letter which may be taken as ${ }_{r}{ }_{\mathrm{r}}^{\mathrm{c}} \mathrm{c}$ will be distinguishable by a frequency that theoretically will be one and two-thirds times as great as the frequency for any ${ }_{\mathrm{nr}}{ }_{5} \theta_{\mathrm{c}}$.

The same reasoning applies to the ${ }_{Y_{c}}^{1} \ldots \theta_{c}{ }^{2340}$ occurrences. Here ${ }_{r} \theta_{c}$ must be the letter $S$, which in the tabulation, should have a frequency greater than that for any other letter, because the basic sequence is $\underline{Y} O N D P$. the same manner, by tabulating all the cases in which $Y_{c}$ appears as the initial letter of a basic sequence and distributing the $1 \mathrm{st}, 2 \mathrm{~d}, 3 \mathrm{~d}, \ldots 25$ th letters after $\mathrm{Y}_{\mathrm{c}}$ into separate frequency tables. In each case, theoretically, the letter which is of highest frequency will be the correct letter, and the process should ultimately yield the complete sequence

YONDSWMAUZXFLQKGXVHRBTECJP

What has been detailed above as regards the YOND... sequence, applies equally well to any other basic cipher-text sequence; any one of them or, in fact, all of them could be reconstructed by the procedure indicated.

In actual practice, however, to have at hand a sufficient volume of text so that there will be 1,000 lines with $A_{c}$ as the initial letter, another 1,000 lines with $B_{c}$ as the initial letter, and so on, up to $Z_{c}$, would mean having about 26,000 lines of text, each 26 letters in length, an enormous it is really not essentiol that the ratio of the frequency of the correct letter to that of any of the incorrect letters in each case be ar areat as 66 is to 39 that is, 17 is to 1 . Ratios of 1.3 to have in actual cases been found to be significant. For evample if the frequency of one particular letter is 15 occurrences, and the frequencies of all other letters yary from zero to approvimately letter is 15 occurrences, and the frequencies of all other letters cary from zero to approsimatey
9 or 10 occurrences, the difference is of sufficient degree to warrant the selection of the letter of 9 or 10 occurrences, the difference is of sufficient degree to warrant the selection of the letter of
highest frequency as being the correct letter. In an actual test, in over 50 percent of the cascs the letter of highest frequency was found to be the correct letter, and in 75 percent of them the correct letter was found to be among the three highest in frequency.

The theoretical minimum number of cases or tabulations necessary to establish ${ }_{r} \theta_{c}$ in actual practice may be regarded as being about 250 , for theoretically in this number the frequency of ${ }_{\mathrm{r}}^{\mathrm{c}} \mathrm{c}$ in actual practice will be 16.5 as against $\frac{233.5}{24}=9.7$ for ${ }_{\mathrm{nr}} \theta_{\mathrm{c}}$; the ratio is approximately 17:10. Now 250 cases of any initial letter in a homogeneous text would require $250 \times 26$ lines of letters, or 6,500 lines. Since each line contains 26 letters, a total of 169,000 letters of text, or approximately 25,000 words would be required. This is considerably below an average day's traffic when the amount of traffic is rather high, as in active operations.

Granting 25,000 words of traffic are available for study it may be said that all the basic sequences applicable to the cipher wheel which acts as CW5 may be reconstructed. (Further details of reconstruction wir be given under sec. Vr.) Having reconstructed the table, the ingle mixed-olvhabet sulstitution ciphers, 20 section $V$ which may single mixed-alphabet substitution ciphers, as explai
readily be solved, as will be illustrated very soon.

## Section Vil

## RECONSTRUCTION OF TABLE OF BASIC CIPHER-TEXT SEQUENCES

Preliminary remarks
Reconstruction of right fixed sequence from two
basic cipher-text sequences
sequences
Reconstruction of MAL5 Reconstruction of table of basic cipher-text sequences from a few lines of cipher text and their equivalent plain-text... 32
30. Preliminary remarks.-It has been shown by the author in previous papers ${ }^{1}$ that two differently mixed primary alphabets from which a series of 26 secondaries are derived can be reconstructed from but two of the derived secondary alphabets. It was shown in paragraph 24, section IV, that the table of basic cipher-text sequences is merely a table of secondary alphinets produced by the sliding of two primary mixed alphabets against each other; one of them is RFS, the other, MAL5. It would seem, therefore, that these secondary alphabets are all interrelated in some manner which ought to permit of the reconstruction of all of them having given only two of them. Only a slight modification of the process is really necessary in order to apply it to the case in hand. First, RFS must be reconstructed; from it and one of the basic cipher-text sequences the entire table can be reconstructed; finally, MAL5 can be reconstructed from RFS and any one of the basic cipher-text sequences.
31. Reconstruction of right fixed sequence from two basic cipher-text sequences.-In order to explain the method, use will be made of the first and second basic sequences of table 1 , assuming that they have been reconstructed as a result of the application of the principles elucidated in the preceding sections.

Sequence 1-..- Y O NDSWMAUZXFLQKGXVHRBTEC JP
Sequence 2---Q Q SF T DP JVAWNYJBLGHEXRKOUMC
It will be noted first of all that each of the two sequences contains only 25 different letters, $X$ being repeated in the YOND. . . sequence and $J$ in the QISF. . . sequence. It is obvious then, that some letter is omitted in each sequence. In the former, $I$ is the letter omitted from the sequence, in the latter, Z. As mentioned once before, and as explained in paragraph 83 of section XV, this repetition phenomenon is unavoidable in this system, and in every case, at least one letter will be repeated, and one will therefore be missing. It is also to be noted that the distance between the repeated letters is always the same in all the sequcnees of the
table of basic cipher-text sequences.

Now superimpose the two sequences, and, hy shifting one relative to the other, make the repeated letters of one ser
other sequence. Thus:
Sequence 1_... YONDSWMAUZXFLQKGYVHRBTECJP (I missing). Sequence 1-... YM N

Construct a chain of equivalents, beginning with any letter of Sequence 1, for example, Y. Thus, $Y U, U D, D Q$, and so on, and then join the pairs eliminating the second occurrence of the
${ }^{1}$ See footriote to p. 20.
common letters. This yields the partial sequence YUDQWSI. This is as far as one gets with this process in this case, because the continuing letter, $I$, is missing from the upper sequence However, if I were present it would seem logical that it would have as its equivalent $Z$, the by adding the letter Z and then proceeding as before. This vields the complete sequence
Y U DQWSIZPOMFVBHLATEXJKNCRG

If examination of this complete sequence of 26 letters is made, it will be found that it contains in regular order, but with a constant difference of three intervals, the letters of RFS. Thus, the RFS proceeds YOEUMXDFVQ. . and the reconstructed sequence proceeds YUDQ. . . and so on

Now, the reconstructed sequence seems to be equivalent to the real RFS, but with a constant difference of three intervals. What is the cause of this difference? Is it not connected with the fact that the initial point at which the measuring circle, CW5, is applied to RFS in the first case is different from that at which it is applied in the second by three intervals. Reference to


Now it is import to hence
列 relation between the elements of the reconstructed RFS should be? The answer is that they do, In the QISF.. . sequence the position occupied by the first occurrence of the repeated letter, J, In the QISF... sequence the position occupied by the first occurrence of the repeated letter, $J$, is three intervals to the left of that occupied by the first occurrence of the repeated letter, X in the YOND sequence, and in order to superimpose the two basic sequences, the lower one was
shifted three letters to the right of its original position. This gives the clue to the correct interval relation between the letters of the reconstructed RFS. If the elements of the latter are therefore distributed over 26 spaces, leaving three intervals between sequent letters of the original reconstructed sequence, then the real RFS will be reconstructed. This yields the sequence
YOEUMXDFJQVKWBNSHCILRZAGPT

It should be added in passing that when the two basic sequences that are available for this reconstruction have their repeated letters at an even number of intervals apart, then a complete anstran hate then a complete united into one sequence. Thus, for example, when the first and seventh basic sequences are selected for experiment, the following results are obtained:


$$
\begin{aligned}
& \text { First half-sequence....- YPARIHNWVJDME } \\
& \text { Second half-sequence..... OT G Z L C S B K Q F X U }
\end{aligned}
$$

Since the second basic sequence had to be displaced 24 letters to the right, in order to bring about coincidence of repeated letters with the first basic sequence in superimposing then the elements of the two half-sequences must be separated by the interval indicated, 24 , yielding the following:

These two half-sequences must now be assembled properly. By this is meant that one of the sequences must be inserted in the spaces presented by the other so as to make the entire sequence coincide with the real RFS. Whether 0 of the second half-sequence should be inserted between $Y$ and $E$ or between $E$ and $M$ or between $M$ and $D$, or between any of the other pairs cannot be determined from the sequences alone.
fuge explained further on.
(See p. 38.)

A complete RFS can only be reconstructed from two sequences whose repeated letters are in positions separated by an odd number of intervals other than thirteen, and any two such sequences will do. It should be added that certain sequences will yield neither a complet equivalent RFS nor two half-sequences. These are the sequences in which the repeated letter in one sequence occupies a position 13 intervals removed from that occupied by the repeated
 how they should be joined is not indicated. Note, for example, the following:
$\begin{array}{lllllllllllllllllllll}\text { Sequence } 1 \text { - }- \text { Y O N D }\end{array}$
Sequence 4-... BNOLETCQHJIRDAPVIGUFYWSMZK

Here $Y=B$ and $B=Y ; O=N$ and $N=0$, and so on. Thus no chain can be constructed. All that one can say is that in the real RFS, $Y$ and $B$ are 13 intervals apart, $O$ and $N$ likewise, and so on.

It is obvious that the cryptanalyst will not know, when he is reconstructing the two basic sequences from a detailed analysis of the cipher text, whether or not he is working upon two sequences such as will permit of reconstructing a complete equivalent RFS after he has finished sequences such as will permit of reconstructing a complete equivalent RFS after he has finished
their construction. If he is fortunate, he may strike it the first time, but if not, he may find their construction. If he is fortunate, he may strike it the first time, but if not, he may find
it necessary to construct several basic sequences before a pair available for a complete reconstruction of the RFS turns up. But having found such a pair, reconstruction is rapid.
32. Reconstruction of entire table of basic cipher-text sequences.- nce two such sequences and the real RFS have been isolated, the entire table of basic cipher-text sequences can speedily be reconstructed in a manner which may best be described by detailed study of table 1

Refer now to the set of sliding alphabets equivalent to CW1 to 5, and note what letter of NAL.5 is concerned in producing Y , the first letter of the YoND... sequence. It is $R$. Now determine what sequence is produced when $S$, the letter following $R$ in NAL5 is concerned keeping AL5 in the same position. The sequence is the 19th shown in table 1, beginning ESFH

Let us superimpose these two sequences:
Sequence 1_... Y ONDSWMAUZXFLQKGXVHRBTECJP
Sequence 19.-.- E S FHBXGMAD JRVWPDKCZNYUIQTO
Now study the sequences formed by oblique lines slanting toward the right, and passing through the superimposed sequences, such as

$$
\begin{array}{llllllll}
\dot{E}^{0} & \dot{S} & \text { N } & \dot{F} & \mathrm{D} & \dot{H} & \mathrm{~S} & \dot{B}
\end{array}
$$

It will be seen that these coincide exactly with the sequences in RFS. By following the order of the letters in the known RFS, and completing the oblique lines of letters, the entire table can speedily be reconstructed. Thus:

TABLE 5.-BASIC CIPHER-TEXT SEQUENCES REARRANGED ACCORDING TO SEQuENCE IN RFS
l Y ONDSWMAUZXFLQKGXVHRBTECJP
19 ESFHBXGMADJRVWPDKCZNYUIQTO
14 H J C N DPXGFQZKBTFWIASOMLVYEU
2 QISFTDPJVAWNYJBLGHEXRKO'UMC
9 LHJYFTQKGBSOQNRPCUDZWEMXIV
5 C Q O J Y V W P NHEVS ZTIMFABUXDLKR
16 VEQOKB TSCUKHAYLXJGNMDFRWZI
3 UVEWNYHIMWCGORDQPS X'FJZBALK
8 KUBSOCLXBIPEZFVTHDJQANGRWM
$18 \mathrm{M} N \mathrm{HE} \mathrm{I} R \mathrm{D} N \mathrm{~L}$ T UAJKYCFQVGSPZBXW
10 S C ULZFSRYMGQWOI JVKPHTANDBX
20 IMRAJHZOXPVBELQ́KWTCYGSFNDH
24 XZGQCAEDTKNURVWBYIOPHJSFCL
4 APVIGUFYWSMZKBNOLETCQHJIRD
25 TKLPMJOBHXAWNSERUYIVCQLZFG
15 WRTXQENCDGB'SHUZMOLKIVRAJPY
12 Z Y DVUSIFPNHCMAXERWLKZGQTOB
130 FKMHLJTS'CIXGDUZBRWAPVYENA
23 JWXCRQYH'ILDPFMANZBGTKOUSGE
21 BDIZVOCLRFTJXGSANPYWEMHPUQ
26 FLAKEIRZJYQDPHGSTOBUXCTMVN 6 RGWUL'ZAQOVFTCPHYENMDIYXKSJ PBMR'AGVEKJYITCOUSXFLODWHQZ NXZGPKUWQOLYIEMHDJREFBCVAT DAPTWMBVEROLUXCFQZUJNIKGYS G'TYRXNKUZERMDIJVAMQSLWPOHF

Thus, given any one of the basic cipher-text sequences, and a knowledge of RFS, the entire table can be reconstructed within a very few minutes
33. Reconstruction of MAL5.-Having reconstructed the table of basic sequences, it is advisable for the next step to reconstruct the mixed component of Alphabet 5. For this, RFS RFS really is, but it is a matter of no consequence, as will be shown.

Taking a sliding strip with the normal alphabet written upon it, and space below it for the insertion of the letters of the mixed component, set it against the reconstructed RFS. Thus:

> AL5...- $\left\{\begin{array}{l}\text { A B C DEFGHIJKLMNOPQRSTUVWXYZ }\end{array}\right.$
> RFS.... $\dot{Y} \dot{O} \dot{E} \dot{U} \dot{M} \dot{X} \dot{D} \dot{F}$ J $\dot{Q} \dot{V} \dot{K} \dot{W} \dot{B} \dot{N} \dot{S} \dot{H} \dot{C} \dot{I} \dot{L} \dot{R} \dot{Z} \dot{A} \dot{G} \dot{P} \dot{T}$

Take the basic sequence YOND and insert the letters which would be successively involved in producing this basic sequence, using any letter, say X , as an initial letter. Thus:

Sliding the strip one space to the left, and considering that the second letter of the basic sequence is 0 , the letter $Y$ must be inserted in the position shown herewith:


When this process is completed, the following sequence results:

\section*{| $A$ | $B$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $X$ | $J$ | $K$ | $L$ | $M$ | $N$ | $O$ | $P$ | $R$ | $S$ | $T$ | $U$ | $W$ | $X$ | $Y$ | $Z$ |}

This sequence is an equivalent mixed component and will produce exactly the same results as the real mixed component, so far as the basic sequences are concerned. In fact, it is the same as the real mixed component so far as the relative values of its sequence of elements are concerned, as is evident when the sequence is "run down" according to the normal alphabet. Thus:

X O YEGJVRPADFMQUHZCIBSNKWTL Y P Z F HK W S Q B E GNRVIAD J C T O L X U M ZQAGILXTRCFHOSWJBEKDUPMYVN ARBHJMYUSDGIPTXKCFLEVQNZWO B S C I K N Z V T E H J Q UY L D G M F W R O A X P C T D J L O A W U FIKRVZMEHNGXSPBYQ DUEKMPBXVGJLSWANFIOHYTQCZR EVFLNQCYWHKMTXBOGJPIZURDAS FW G M O R D Z X I LN UYCPHKQ JAVSEBT GXHNPSEAYJMOVZDQILRKBWTFCU HYIOQTFBZKNPWAERJMSLCXUGDV I Z J PRUGCALOQXBFSKNTMDYVHEW JAKQSVHDBMPRYCGTLOUNEZWIFX KBLRTWIECNQSZDHUMPVOFAXJGY LCMSUXJFDORTAEIVNQWPGBYKHZ M D N T V Y K G E P S U B F J W O R X Q H C Z L I A NEOUWZLHFQTVCGKXPSYRIDAMJB OFPVXAMIGRUWDHLYQTZSJEBNKC P G Q W Y B N J H S V X E I M Z R U A TK F C O L D Q HRXZCOKITWYFJNASVBULGDPME $\rightarrow$ RISYADPLJUXZGKOBTWCVMHEQNF S J T Z B E Q M K V Y A H L P C UXDWNIFROG TKUACFRNLWZBIMQDVYEXOJGSPH ULVBDGSOMXACJNREWZFYPKHTQI VMWCEHTPNYBDKOSFXAGZQLIURJ W N X D F I U Q O Z C E L P T G Y B H A R M J V S K
Note the generatrix beginning RISY... , and compare it with the real MAL5. It is idenNote the generatrix beginning RISY...
tical with it, except as regards its initial letter
The process described above, for reconstructing MAL5, may be used to determine how the two equal halves of a reconstructed RFS should be united (see p. 34) and at the same time reconstruct IAls. By employing each hal-sequence of RFs in conien epher-text sequy process of reconstruction to the half-sequence
in connection with the YOND... basic sequence, the following partial MAL5 is constructed (using X as a starting point)

Applying a similar process to the other half-sequence,
O . U . X . F . Q . K . B . S . C . L . Z . G . T
still in connection with the YOND. . . basic sequence, the following partial MAL5 is constructed:

$$
\begin{aligned}
& \text { A B C D E F GHI J K L M N O P Q R S T U V W X Y Z } \\
& \text { Y G J }
\end{aligned}
$$

These two partial MAL5 sequences are now to be united.
Now it will be found that there is one and only one way in which these two sequences may be united properly, so as to make one sequence-they must be "dovetailed" into each other, the This positions in the first half being exactly filled by letters in the second half, and vice versa This union is as follows, where the letters underlined with dots belong to the one partial sequence mose underlined with dashes belong to the other partial sequence

## X O Y E G J V VR P A D FM Q U H Z C I B S N K WT

This sequence is complete, and contains no repetitions. Comparison with the MAL: sequence derived above will show their identity
With MAL5 completed, it is only a simple step to unite the two half-sequences of the RFS, has constructed
It is obvious, of course, that if RFS and MAL5 are known sequences to start with (say as the result of espionage), then not even one of the basic sequences need be constructed from the laborious process of text analysis. All
two known sequences, RFS and MAL5

When the entire table of basic sequences has been reconstructed the cryptanalyst is in a posicion to decompose the lines of cipher text into the elements of a series of single mixed-alphabet substitution ciphers, which can be solved in a comparatively short time. It will be unnecessary demonstrate the process at this point as it will come up again later in practically the same form. 34. Reconstruction of table of basic cipher-text sequences from a few lines of cipher text and their equivalent plain text. - It should be apparent from what has preceded, that if a few lines of cipher text with their letter-for-letter decipherments are at hand, it would be a very easy matter to reconstruct two basic sequences from which the entire table could then be derived. Such a case of liaving the cipher text with its plain text equivalent is not at all rare in practice, where such blunders as repeating a message in clear, after it has been transmitted in cipher or the reverse, sometimes occur. Or often, a plain-text dispatch is captured, whereupon it can be compared with its cryptographic form; or a paraphrased version of a dispatch is given sary to establish the entire table of basic sequences, why five or six lines of 26 ctters are necesproduced by the same machine is at once reduced

Section Vili

## general observations

Résumé of preceding analysis
${ }_{35}$
Fundamental assumption for military cryptog.
.
35. Résumé
of preceding analysis.-It has been shown thus far how the table of basic the the text of dispatches. At least two basic sequences are necessary for the reconstruction, If MAL5 and the RFS were previously known it would be unnecessary to reconstruct one of the If MAL5 and the RFS were previously known it woud be unnecsery
36. Fund milat assumption
36. Fundamental assurn - A fundamental assumption with regard to the use of any device for cryptographic purposes in military operations is this: It must be granted that the enemy cryptanalysts are in possession of full knowledge as to the mechan-
ics of the device and, in fact sooner or later, come into possession of one, by capture, or even legitimately, by purchase, in the case of machines for sale upon the open market. Hence, in the case of the present machine, even if it be granted that the wiring of all the machines for use in the military service be absolutely secret to begin with, it must be assumed that the enemy already is thoroughly familiar with the mechanico-electrical operation of the machine, or will soon capture one or more of the machines upon the field of operations, and will thus learn the secret wiring
37. Deductions from fundamental assumption.-Now it goes without saying that the capture or loss of a single machime would necessitate an immediate change in the wiring of all the machines in service. This conld be accomplished in one of two ways. (1) The operators in the field could change the wiring according to directions from higher headquarters, or (2) new cipher wheels could be issued by higher headquarters. It is the opinion of the writer that the former case may be ruled out at once, for it would be entirely impractical in the ficld. A single error in wiring ( 182 connections must be established) would make all messages unintelligible; the time necessary for the change to be made would hardly be available, nor would the personncl with the requisite training always be available. The latter case, where new wheels are distributed from a central office is more feasible, but even in this case there are many difficuties, as in the Thater of Oprations Grating the end case hower, the wiving or civerits establisher
 the left and right fived sequences, would still be unchanged, unless directions for the change em nate from the central office. Now there are 52 connections established in the rear plate; it a practical certainty that many errors would be made by troops in the field working under diff culties, from written instructions. The process cannot be done in less than two hours, and skillful fingers are necessary. Practically, therefore, the change could not be made by troops in the field, and the wiring in the rear would remain permanent. The capture of one machine will disclose to the enemy the two important sequences, the LFS and RFS.

Having RFS at hand, it is apparent from what has gone before, that only a single basic sequence would be necessary in order to reconstruct the entire table of basic sequences. Or if MAL5 could be reconstructed by some process of analysis of the cipher text itself, not even one of the basic cipher-text sequences would be necessary.

The process whereby one of the basic sequences can be reconstructed from an analysis of the cipher text itself has been explained in detail. If a knowledge of the RFS is assumed, ought not the process be rendered more easy? Common sense would lead one to answer in the affirmative. Perhaps one could reconstruct MAL5 directly from an analysis of a comparatively larger volume of text required to build up one of the basic sequences first. This possibility forms the subject of the next section.

Section IX

## RECONSTRUCTION OF ALPHABET 5

Further analysis of the nature of Alphabet 5 . Use of CAL5
Uelation existi
Relation existing between CAL5 and the table of A dilemma.
Application of mathematical theory

Par.
38
Theore
 Explanation of the discrepancy between mathematical theory and actual data-................. ${ }_{42}^{41} \begin{aligned} & \text { Necessity for additional tables_-................... } \\ & \text { Procedure after MC AL5 has been reconstructed.- }\end{aligned}$
38. Further analysis of the nature of Alphabet 5.-If Alphabet 5 is set against RFS at the initial point, so that A of NAL5 is above T (the initial letter of RFS), it will be seen that Y, the first letter of the basic cipher-text sequence YOND. . . is under B of NAL5. Sliding Alphabet first letter of the basic cipher-text sequence YOND... is under B of NAL5. Siding Alphab
5 one interval to the left, 0 , the second letter of the basic sequence YOND... is under D of 5 one interval to the left, 0 , the second letter of the basic sequence YOND... is under D of
NAL5, and so on. The whole sequence of letters of NAL5, that are successively concerned in NAL5, and so on. The whole sequence of letters of NAL5, that are successively concerned in
the production of the YOND. . sequence, as seen by noting the letters above the successive letters of the YOND... basic sequence, as Alphabet 5 is slid to the left is as follows:

BDRKUSLEMFQTGXANWCJOIVZPHY
Now take another basic cipher text sequence, say the second one of table 1, QISF. repeat this process and write down the sequence of letters given by noting the letters of NALS that appear above the successive letters of the QISF ..
basic sequence as Alphabet 5 is to the left. It is as follows:

KUSLEMFQTGXANWCJOIVZPHYBDR
Comparing the two NAL5 sequences obtained from these two basic sequences it is noted that they are the same sequence-merely their initial points are different. In fact, one and only one sequence results when all the basic cipher-text sequences are treated in the same way What is this sequence? It seems to be some fundamental sequence that has not been encountered before this. It does not appear as any one of the alphabets of the diagrams given so far

Consider once more the juxtaposition of Alphabet 5 and RFS for the YOND
. basic sequence and find the point at which the current enters CW5 in order to produce this basic sequence. It is at the point designated by the arrow in the following diagram: the eighteenth contact of RS5:


The LHC of CW5 at which the current enters to produce the Y is R . But the fact that $Y$ is the cipher resultant, and the fact that the letter of NAL5 that is over $Y$ in the RFS is B may when taken together be regarded as equivalent to assuming that $R$ is converted into $B$ and $B$ is then converted into $Y$. In short, it would appear as though an " $R$ current" is converted into a "B current" by the wiring of CW5, and further, that at the position indicated, the "B current" finally emerges as a "Y current." Hence, considered independently, and only
with regard to the cipher mechanics involved, Alphabet 5 might be written, so far as only $R$ and its conversion-equivalent are concerned, as follows:
AL5...- $\left\{\begin{array}{l}A, B D E F G H I J K L M N O P Q \stackrel{\downarrow}{R} S T U V W X Y Z A B C\end{array}\right.$
RFS_... $\dot{T} \dot{Y} \dot{O} \dot{E} \dot{U} \dot{M} \dot{X} \dot{D} \dot{F} \dot{J} \dot{Q} \dot{V}$ K $\dot{W} \dot{B} \dot{N} \dot{S}{ }_{H}^{B} \dot{C} \dot{I} \dot{L} \dot{R} \dot{Z} \dot{A} \dot{G} \dot{P}$
Now repeat the process for the second letter of the YOND. . . basic sequence, having slid Alphabet 5 one space to the left.

RFS_ $\dot{T} \dot{Y} \dot{O} \dot{E} \dot{U} \dot{M} \dot{X} \dot{D} \dot{F} \dot{J} \dot{Q} \dot{V} \dot{K} \dot{W} \dot{B} \dot{N} \dot{S} \dot{H} \dot{C} \dot{I} \dot{L} \dot{R} \dot{Z} \dot{A} \dot{G} \dot{P}$
Here an "S current" is converted into a "D current": the latter, into an " 0 current." Hence, the placement of $D$ is as follows:

The next letter, N , of the YOND... basic sequence, is the result of the interaction of $\mathrm{T}, \mathrm{R}$, and N , as follows:

ALS $\left\{\begin{array}{l}A B C D E F G H I J K L M N O P Q R S T U V W X Y Z A B C .\end{array}\right.$
RFS • $\dot{T} \dot{Y} \dot{O} \dot{E} \dot{U} \dot{M} \dot{X} \dot{D} \dot{F} \dot{J} \dot{Q} \dot{V} \dot{K} \dot{W} \dot{B} \dot{N} \dot{S} \dot{H} \dot{C} \dot{I} \dot{L} \dot{R} \dot{Z} \dot{A} \dot{G} \dot{P}$
from which follows:

RFS $\quad$ T $\dot{Y} \dot{O} \dot{E} \dot{U} \dot{M} \dot{X} \dot{D} \dot{F} \dot{J} \dot{Q} \dot{V} \dot{K} \dot{W} \dot{B} \dot{N} \dot{S} H \dot{C} \dot{I} \dot{L} \dot{R} \dot{Z} \dot{A} \dot{G} \dot{P}$
Combine the conversion results of $\mathrm{Y}, \mathrm{O}$, and N into one sequence:

The mysterious sequence which started this train of reasoning began with BDR . the process, the identity of the sequence of conversion-equivalents with the sequence $B D R K$. will be established, and so far as the cipher mechanics of CW5 are concerned, Alphabet 5 may be written as follows:

A B C D E F G H I J K M N O

The Alphabet 5 which has heretofore been used, and the new or conversion Alphabet 5 may be placed in juxtaposition for study
$\left.\begin{array}{llllllllllllllllllllll}A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & R & S & T & U & V & X \\ F & R & Y & Z \\ \hline\end{array}\right\}$ Alphabet 5 .
FRISYADPLJUXZGKOBTWCVMHEQN
ABCDEFGHIJKLMNOPQRSTUVWXYZ Converted

These two alphabets manifest the simple enciphering-deciphering relationship of one and the same mixed alphabet: If the enciphering alphabet is at hand, the deciphering alphabet can be constructed, and vice versa, just as is the case with any ordinary mived alphabet used in cryptography.

For ease in reference, the letter C will be used as a prefix to an alphabet designation to indicate that it is the converted equivalent of the real alphabet. Thus, CAL5 refers to the second of the two alphabets above, and NCAL5 refers to its normal component, MCAL5, to its mixed component.

In this case, either AL5 or CAL5 can be used in encipherment. In using the real Alphabet 5 (AL5) one proceeds from a letter in the normal component to the same letter in the mixed component and then takes the letter directly under it in the right fixed sequence (RFS). In asing the CAL5 one takes as the cipher equivalent of a letter in its normal component that letter which is directly under it in its mixed component, and then notes the letter of RFS above which the cipher equivalent, as it is located in the normal component, falls. Thus, for example, for Y of the YOND... sequence, the chain is as follows: R of NCAL5 is converted into B of NAL5; but $B$ of NCAL5 is now opposite Y of RFS. For the second letter, 0 , of the YOND... sequence, the chain is as follows: S of NCAL5 is converted into D of NCAL5; but D of NCAL5 is now opposite 0 of RFS. For the third letter, N, of the YOND... sequence, the chain is initiated with the letter T of NC.AL5; for the fourth letter, D, of the YOND. . sequence, the chain is initia
with the letter $U$ of NCAL5, and so on, according to the sequence of the normal alphabet.
Simiar relations will be found to obtain with respect to the other four alphabecs, converted quivalent e is consthent.
merte ored in a reated as converted equivalent alphabets, and in a subsequent section the relation between real alphabets and the converted equivalents will be demonstrated

The set of converted alphabets is as follows:
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ GADBOCTKNOZXIWHFQYJVPMELSRGADBOCTKNUZXIWHFQYJVPMELSR ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ IZNCTKUDPJEVOWLFHXSMGQAYBRIZNCTKUDPJEVOWLFHXSMGQAYBR ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ PJXFWLTAUGYBMHROVNCKSEQIZDPJXFWLTAUGYBMHROVNCKSEQIZD ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ FLVARGWCMQBXNYIOTJUPSKEDHZFLVARGWCMQBXNYIOTJUPSKEDHZ
ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMNOPQRSTUVWXYZ FQTGXANWCJOIVZPHYBDRKUSLEMFQTGXANWCJOIVZPHYBDRKUSLEM

CAL 1
CAL 2.
CAL 3
CAL 4
CAL 5.
39. Use of CAL5.-Returning now to the example of encipherment using the phrase "THE ELEMENTS OF THE SCIENCE OF", let those letters of NCAL5 which are above the tetters of RFS and which constitute the cipher letters of the cryptogram in each case be set down The enciphered text is as follows:
IU U JUV JPFP JSCLVIKSDBMZDJSK

The first letter of the cryptogram is $I_{c}$. Referring to the sliding strips it will be seen that $I_{c}$ as it occurs in RFS is directly under T of NCAL5. Thus:
(The identity of T , the plain-text letter, with T , the letter of NCAL5 directly over I, the cipher letter is, of course, merely a coincidence.

Alphabet 5 must be slid one space to the left, for the next letter
ABCDEFGHIJKLMNOPQRSTUVWXYZABC
TYOEUMXDFJQVKWBNSHCILRZAGP
The second cipher letter is $U$, and this letter of RFS is now beneath $F$ of NCAL5. Again Alphabet 5 is slid to the left, and the letter in NCAL5 above 0 (the third cipher letter) in RFS is found to be E. This process continued, results in the following:
Plain
THEELEMENTSOFTHESCIENCEOFC
Cipher
Ctters of NCAL --............. I UO JUVJPFPJSCLVIKSDBMZDJSK

Ih
The letters of the sequence TFEM ... may be designated as the cipher-text equivalents of the normal component of converted Alphabet 5; this long designation will hereafter be referred to as the NCAL5 $5_{\mathrm{c}}$ equivalents.

Now apply MCAL5 to these NCAL5 $\mathrm{c}_{\mathrm{c}}$ equivalents (TFEM...) making T the first letter coincide with T of MCAL5 and underline the coincidences. Thus:

TFEMIQPGQITBEHZICHZHZRDGOL
TGXANWCSOIVZPHYBDRKUSLEMFQ
It is noted that the 1st, 10th, and 14th letters coincide.
Now apply MCAL5 to the NCAL5 equivalents so that $F$, the second letter of the latter coincides with F of the former. Thus:

$$
\begin{aligned}
& \text { TFEMIQPGQITBEHZICHZHZRDGOL} \\
& M \underline{F} \text { Q GXANWC J OIVZPHYBDRKUSLE }
\end{aligned}
$$

It is noted that the 2 d and 15 th letters coincide.
If the same process is applied with respect to the third letter of the series of NCAL5 equivalents, coincidences of the 3d, 4th, 6th, 8th, 16 th, 20 th , and 23 d letters are noted. When the lents, coincidences of the $3 \mathrm{~d}, 4 \mathrm{th}, 6 \mathrm{th}, 8 \mathrm{th}, 16 \mathrm{th}, 20 \mathrm{th}$, and 23 d letters are noted. When the dentical numbers indicate coincidences yielded by the successive applications of MCAL5 to NCAL 5 c equivalents:

I U O J U V J P F P J C C L
T F E M I Q P G Q I T

Now apply the numerically distributed sequence to the plain text. Thus: THEELEMENTSOFTHESCIENCEOFC T F E M I Q P G Q I T B E H Z I I C H Z H H Z R D G O L

It will be noted that similarly numbered letters of the $N C A L 5_{0}$, sequence here also indicate coincidences of plain-text letters. This is not an isolated phenomenon applying only to the single line of cipher text under consideration but is a fundamental and general principle that applies to all lines of the cipher text produced by this machine. (Let the reader prove this by enciphering a phrase and applying the process indicated above.) In other words, if MCAL5 were at hand, and if the cipher text can be converted into its $\mathrm{NCAL5}_{\mathrm{c}}$ equivalents through a knowledge of the RFS, then all letters of the cipher text representing identical letters in every line of the plaintext of 26 letters of the cipher text can be found and indicated by properly assigned reference numbers. Each line of cipher-text will thus be decomposed into the elements of a sing the mixed-alphabet substitution cipher, just as was the case in the preceding method usio the table of basic cipher-text sequences. Having assumed a knowledge of RFS in this section, the
unknown factor is MCAL5. If that can be reconstructed by analysis, the problem is solved. 40. Relation existing between CAL5 and the table of basic cipher-text sequences. - Whereas
in section VI, all 26 basic sequences are necessary to effect this decomposition of each line of cipher text into its sets of identical elements, in this case one and only one sequence is necessary. Take, for example, the two letters F...X of MCAL5. Whenever any two identical plain-text letters separated by four intervals are so enciphered that the NCAL5 equivalent of the first letter is F , the $\mathrm{NCAL5}_{\mathrm{c}}$ equivalent of the second cipher letter will he X , no matter where this pair of plain-text letters happens to fall within the line of 26 letters. Why this must be so can readily be seen by referring to an actual encipherment

Consider CAL5 in the position indicated below, assume that the first E of plain-text ${ }^{12345}$..E is being enciphered at the seventeenth displacement of CW5 from its initial point, and assume that the electric current enters CW5 at the point indicated by the arrow. The cipher resultant will be $B$

NCAL5.... ... PQRSTUVWXYZÁBCDEFGHIJKLMNOPQRST... MCAL5.... . . . HYBDRKUSLEMMQTGXANWCJOIVZPHYBDR... RFS TYOEUMXDFJQVKWBNSHCILRZAGP

For the second E of E...E, the sliding alphabet will be in this position, the twenty-first displacement of CW5:
NCAL5.... . . . TUVWXYZABCDE FGHIJKLMNOPQRSTUVWX...
MCAL5.... . . . RKUSLEMFQTGXANWCJOIVZPHYBDRKUSL...
RFS...- TYOEUMXDFJQVKWBNSHCILRZAGP
The cipher resultant will be 0 .
In the first case the current entered CW5 at the LHC of A, in the second case it entered at the LHC of E. From A to E in the normal aiphabet there is an interval of four letters: from 5973-34-4
the first E of the plain-text $\mathrm{E}^{12345}$. E to the second, there happens to be an equivalent interval The first phenomenon (distance from A to E in the normal alphabet) is a constant one; the second, is an accidental one, and the two phenomena are, of course, not causally related; they merely coincide as a matter of chance. But in the first instance, the current entering CW5 as an E current was changed into an F current, but F was then opposite B of the RFS; in the second instance, the current entering CW5 as an E current was changed into an $X$ current, but $X$ was then opposite 0 of the RFS. B and 0 are therefore causally related through the intermediacy of the measuring circle, CW 5 , that is, specifically through the intermediacy of the distance from F to X on MAL5, which happens to be four intervals. Whatever the cipher resultants be, so long as the two letters F...X of MAL5, are the ones that are involved in the encipherment of two letters four intervals remored from each other, and so long as no displacement of $\mathrm{CW} 1,2$,
3, or 4 has occurred between the two encipherments, these different cipher resultants will 3, or 4 has occurred between the two encipherments, these different cipher resultants will cipher resultants for this measuring interval $\mathrm{F}^{12345} \ldots$. . They are as follows:

| Position of CW5 | Cipher resultants | $\underset{\substack{\text { Pusition } \\ \text { CW5 }}}{ }$ | Cipher resultants |
| :---: | :---: | :---: | :---: |
| 1... 5 | M. . . I | 14... 18 | C... X |
| 2... 6 | U...C | 15... 19 | H. . . ${ }^{\text {M }}$ |
| 3... 7 | E...H | 16... 20 | S...U |
| 4... 8 | 0...S | 17... 21 | N...E |
| 5... 9 | Y...N | 18... 22 | B. . . 0 |
| 6... 10 | T... ${ }^{\text {B }}$ | 19... 23 | W... ${ }^{\text {P }}$ |
| 7...11 | P...W | 20... 24 | K...T |
| 8...12 | G.... ${ }^{\text {K }}$ | 21...25 | V....P |
| 9... 13 | A...V | 22... 26 | Q...G |
| 10... 14 | Z...Q | 23... 1 | J...A |
| 11... 15 | R...J | 24... 2 | F...Z |
| 12... 16 | L...F | 25... 3 | D. . .R |
| 13... 17 | I... D | 26... 4 | X...L |

Similarly, there can be only 24 other sets of such 26 pairs of resultants for the distances between F and all the other letters on MAL5, for there are only 24 intervals between F and the other 25 letters of the alphabet, viz, those between $F$ and $A, F$ and $B, F$ and $C$, and so on. For each letter of the alphabet there will be a total of 25 sets of 26 pairs of cipher resultants, yielding a grand total of 650 sets of 26 pairs. These, with a peculiar arrangement among themselves, form the table of basic cipher-text sequences. For example, M. . . I will be found in the 1 st and 5 th positions of the 18 th sequence of table $1 ; U^{12345} \ldots$. $^{2}$ will be found in the 2 d and 6 th positions of the 8 th sequence; E...H, in the 3rd and 7 th positions of the third sequence, and so on. Table 1 has 676 elements, capable of forming 676 sets of 26 pairs, but examination will show that there are two identical letters in each sequence and one letter always missing. This has been referred to before.

If the interval relations between one letter and all the other letters in MCAL5 can be established, that is all that is necessary to establish the whole mixed component, for then the position of each letter in the mixed component relative to all the letters can be definitely fixed, letters. For example, having established the sequence FQIGX in a hypothetical MCAL5,
he interval relations between $F$ and $Q, F$ and $T, F$ and $G, F$ and $X, Q$ and $T, Q$ and $G, Q$ and $X$, and so on are automatically given.

Now then, can the interval relations between any one letter and all other letters in an unknown MCAL5 be established by an analysis of the cipher text alone? In other words, assuming a knowledge of RFS can MCAL5 be reconstructed very easily from the cipher text itself? This is the kernel of the problem.
41. A dilemma.- It has been shown how the letters of each line of cipher text can be conerted into their $\mathrm{NCAL5}_{\mathrm{c}}$ equivalents through a knowledge of RFS. If in each line there were some indication that would lead to identifying those conversion equiralents which represent encipher and of the same letter, then obviously MCAL5 could be quickly constructed. For exanple, and position of identities were arailable, then one would say that MC.AL5 is made up of the partial sequences shown:

arse prial sequences by the principle of direct symmetry of position, the following result is obtained:

T G . . . C . OI . Z . H . B DR . . L EMFQ
Over half of the sequence has been reconstructed from but one line of cipher text. The reconstruction of MCAL5 resolves itself therefore into the problem of finding merely those letter's in each ine of the cipher horizontal lines. But, unfortunately, it would seem that is, simply loca
 dilemma be solved?
42. Application of mathematical theory.-Keference is now made to the mathematical 42. Appling the effects of repetition and nonrepetition as set forth in section VI.
theory concerning the was shown therein the basic cipher-text sequences can be reconstructed by a mathetical analysis based upon the mere existence of repetition, but in such reconstruction each basic sequence represents a separate problem and the data pertaining to each one must be carefully isolated from those pertaining to all other basic sequences. Hence in reconstructing basic sequences by the application of the mathematical theory it was necessary that the reconstruction be based upon the initial letter of each particular basic sequence being reconstach, and a total of 26 such separate reconstructions, corresponding to the 26 individual basic seffuences, is possible. It will be noted that there are absolutely no repectitions of letters winh acter tabie 1, and a thorough understanding of the machine will show why imposib.e. A ictter, $\theta$, in any individual basic sequence has as ths stecessors lan occupies in that basic sequence. For example, take the first basic sequence, YON...CJP

In compiling the frequency tables necessary to reconstruct this sequence by the mathematical theory of repetition only cases in which $Y$ appears as the rery first letter in the line can be used to buid up the sequence, a $Y$ in any ore Y in hence the data based upon $Y$ in the first position cannot be linked with those based upon that is different from all other sequences in which 0 appears in any other position. Hence it is clear that a great deal of text is necessary to permit of reconstructing by this method a basic sequence, and as was shown in the last two paragraphs of section VI text consisting of approvimately 25,000 words must be available for analysis before any basic cipher-text sequences can be reconstructed.

But in the case of the MCAL5 one and only one sequence is involved, and there will therefore be but one case of $Y^{12345}$. $\theta$ regardless of where the $Y$ occurs in each line. Hence all the data with respect to the Y...日, for example, can be placed in a single table, and all the data with respect to all other pairs of letters separated by the same interval, can be grouped into the same table. Thus, for example, a single frequency table which shows all the pairs of the formula $\theta_{1} \theta_{2}$ (sequent letters) would be made regardless of where $\theta_{1}$ is located in each line of 26 cipher-text-normal-alphabet-converted-equivalents; another table would be made for pairs of the formula $\theta_{1} \theta_{2}{ }_{2}$ (i.e., separated by two intervals); another for the formula $\theta_{1} \theta_{2}$ (i.e., separated by three intervals), and so on up to those of the formula $\theta_{1} \theta_{2}$. If a sufficient amount of text were available, 25 such individual or separated tabulations based solely upon the cases in which $A_{c}$ is selected as $\theta_{1}$ and showing what $\theta_{2}$ is for all the 25 positions after $A_{\mathrm{c}}$, would be all that would be required. In reality only 13 tabulations would be necessary for $\mathrm{A} \theta_{2}$ would be the same as $\theta_{1} \mathrm{~A}, \mathrm{~A} \theta_{2}$ would be the same as $\theta_{1} \mathrm{~A}$, and so on. These 13 tabulations can really be grouped into one table in which all the data for all cases of ${ }^{1234 \ldots} \ldots \theta_{2}$ will be included.

The mathematical theory as here applied would be as follows:
In every 1,000 pairs of $\mathrm{NCAL}_{\mathrm{c}}$ equivalents separated by a constant interval, but taken from the same horizontal line of text, there will be 66 cases in which both members of the pair are the NCAL5 equivalents of the same plain-text letter; there will be 934 cases in which they are the NCAL5 equivalents of two different plain-text letters. Let the members of a pair be represented by the symbols $\theta_{1}$ and $\theta_{3}$. In the $\delta 6$ rases of repetition, $\theta_{2}$ will always be the same letter; in the 934 cases of nonrepetition $\theta_{2}$ can be any one of 24 other letters, thus giving each of the 24 letters an average frequency of 39 . Reasoning conversely, therefore, when the $\theta_{2}^{\prime}$ 's of 1,000 cases of $\theta_{1} \theta_{2}$ are distributed over a frequency table, that $f$ only 30 will be the NCAS of only 39, will be the NCAL5 equivalent of the some letter of which $\theta_{1}$ is the NCAL5 equivalent.

For example, in a tabulation of the cases of the formula $\mathrm{G} \theta_{2}$ among the $\mathrm{NCAL5}_{\mathrm{c}}$ equivalent of approximately 2,000 letters of cipher text, enciphered by the alphabets used in this demon stration, the following distribution of $69 \theta_{2}$ 's was obtained:


Reference to MCAL5 (p. 43) shows that the sequence G...W is correct. Here only a total of 69 obserrations are recorded, and yet the correct letter, W, manifested itself. Had 1,000 cases been observed there is absolutely no doubt about what the result would have been. The whole table for the fourth interval after $\mathrm{A}, \mathrm{B}, \mathrm{C}, \ldots, \mathrm{Z}\left(\theta_{1} \theta_{2}\right)$ based upon only the 2,000 letters of ipher text mentioned above, is shown in the accompanying table 7

Even in this small number of observations the actual results are in fair conformity with the heoretical expectancy. In each case the correct letter is indicated by a circle. In certain cases he correct letter is among the very lowest in frequency, as for example in the case of the $H$ distribution, but in 15 cases the correct letter is either the highest or second highest in frequency.

TABLE 7.-DISTRIbution of $\begin{gathered}1 \\ \theta_{1} \\ \theta_{2}\end{gathered}$ IN 2,000 Letters of text

43. Theoretical considerations relative to tables to be constructed.-The accompanying table was based upon a study of pairs of the formula $\theta_{1} \theta_{2}$ and the question may be raised as to whether one and only one table is sufficient to permit of a reconstruction of MCAL5, or whether several tables are necessary. If one table will suffice, upon what interval relationship of pairs several tables are necessary. If one table will suffice, upon what interval relationship of pairs
should it be based for the most conclusive results? Theoretical consideration will show that if a large volume of text is at hand, one table will suffice if the interval used is an odd interval, other than 13. If the interval is even, the best that can be expected is two half-sequences of 13 letters each. For example, suppose the fourth interval table is made, and suppose further that the table covers a sufficiently large number of observations so that the results in each case may be regarded as positive in that the letter of highest frequency will always be the correct letter. Then by constructing a chain of the letters of highest frequency, two sequences of 13 letters in MCAL5 can be established. For example, in this case suppose that in the A . . . $\theta_{2}$ distri-

 highest in frequency for $\theta_{2}$; in the Z . . . $\theta_{2}$ distribution $B$ shows up as highest in frequency for $\stackrel{\theta}{2}_{2}$, and so on, this chain establishes itself: A. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | seventh placement, $G$, falls into this position:

The next placement, $W$, would fall into this position:

Continuing the process a sequence of 13 letters in MCAL5 may thus be established from one table. The sccond half-sequence would be obtained by starting with a letter not found in the first set. However, if the interval had been odd and not 13 , the chain would have continued until all 26 letters had been properly placed. As a general rule, it may be said that it will be safer to compile several tables which will be mutually corroborative, as explained in paragraph 45.

However, it is interesting to see whether or not, from theoretical considerations alone, there should be a certain single table which would give more conclusive results than any other single table. For example, if only one table were to be construct

The mathematical theory of repetition and nonrepetition, as developed in this paper, postulates that 66 out of 1,000 observations of any pair of letters will be repetitions, the remaining 934 will not be repetitions. In an actual test of the theory upon plain-text, the data shown in table 8 were obtained. Column 1 applies to observations made upon letters that were sequent, that is, separated by a single interval, formula $\theta_{1} \theta_{2}$; column 2 applies to observations made upon letters separated by two intervals, formula $\theta_{1} \theta_{2}$, and so on by increasing intervals.

Table 8
Interval
Actual repetitions

44. Explanation of the discrepancy between mathematical the ory and actual data.-Accord ing to the mathematical theory the tabulations in table 8 should all be practically equal but it is seen that the actual results differ from the theoretical expectancy to a slight degree. These discrepancies are due to two causes: (1) An insufficient number of observations, and (2) whereas the mathematical theory postulates a purely random selection based upon a thorough mixture of the letters of the text, actually, letters forming inteligible text show a marked degree of association which tends to distort he theoretical expectancy. That is, for example, E tends to
 fllible, , such as the mathematical theory postulates. The approximation to the theoretical is closest
in the case of pairs of the formula $\theta_{1} \theta_{2}$, as shown in table 8. It is extremely probable that if an extensive study were made of this point, the relation between the interval separating repetitions and the frequency of the repetitions could be expressed in the form of a curve of the following nature:


The reason for this is not difficult to see. The closer together the members of any $\theta_{1} \theta_{2}$ pair stand in such intelligible text, the more likely is it that the natural affinities of letters will manifest themselves so as to distort the theoretical expectancies based upon a purely random selection conversely, the further apart these members are, the less likely is it that such natural affinitie will manifest themselves. Hence, it should follow theoretically, that tabulations based upon pairs of formulae greater than say $\theta_{1} \theta_{2}$ should most closely approximate the theoretically expected results because the intervals between each pair are great enough to overcome or suppress the natural affinities of letters constituting clear text.
45. Necessity for additional tables.-Tables based upon other intervals may be necessary corroborate results obtained from the study of but one table. For example, having determined from a 4 lli interval table the sequence A...J...Z...B...., a table based upon the 8 th interval should show A......Z; J......B; and so on. Thus, corroboration of placements can be obtained. With the definite placement of each letter, the possibilitics for the placement f the remaining leters definite placement of each letter, the possibitics for the placest of the remaining low with for TCAL 5 becomes progressively easier and easier, providing no mistales are made due to insuff CAL5 becomes progressively eas
46. Procedure after MC
6. Procedure after MCAL5 has been reconstructed.-Once MCAL5 has been reconstructed one can proceed immediately to underline by distinctive colors in each line of text the
 structed MCAL5 and RFS in the manner illustrated in paragraph 32, section VII. Then one
can underline the cipher letters themselves, in distinctive colors, to represent identical plain-text letters. From this point on, one is confronted with a slightly modified form of a single mixed alphabet substitution cipher as stated in the previous sections. With certain short cuts to be explained later, solution of all messages is then readily achieved.
${ }^{1}$ Instead of underlining, by distinctive colors, NCAL5 equivalents representing identical plain-text letters one may designate the identities by some other method, for example, by the numeration method shown in parascience of cryptanalysis" are assigned identical numbers.

## Section X

## PRACTICAL APPLICATION OF PRINCIPLES

Nature of the test
Arrangement of dispatches............
Finding NCAL5 equivalents

| Par. |  |
| :--- | :--- |
| 47 | Study of the tables and reconstruction of MCAL5- |
| 48 | Reconstructing the table of basic cipher-text | inding NCAL5 ${ }_{c}$ equivalents $\qquad$ sequences

Solution of $t$ os the fir e of cipher text52
53
47. Nature of the test.-Attention will now be directed to the application of the foregoing principles to the analysis of an actual problem. The Code and Signal Section of the Navy by the author were made, presented the writer with a series of ten cipher messarges enciphered y a machine in which they had changed the wiring of the cipher wheels so that this wiring was entirely secret, whe for the the whing of the LFS nd RFS remained the same as bare and was, and RFS remained the same as before and was, of course, known to the writer. ${ }^{1}$ The text of
the ten messages and the key settings applying to the wheels, except the $2 d$ and 4th cipher wheels, the ten messages and the key settings applying to the wheels, except the 2d and 4th cipher wheels,
the settings of which were different for each dispatch and were hept secret from the uriter, are given the settings of which were different for each dispatch and were kept secret from the uriter, are given
in the appendix. The theory behind the secrecy as regards cipher wheels two and four is that in the appendix. The theory behind the secrecy as regards cipher wheels two and four is that all chances of two or more dispatches from different stations being enciphered by exactly the same key.
48. Arrangement of dispatches.-The dispatches as presented for analysis were writte out in lines of 26 letters each corresponding to the initial position of CW5 at its encipherment For example, Dispatch No. 1 was enciphered by the key AGRAM. This means that the initial apparent setting was as follows:

LAW CW1 CW2 CW3 CW4 CW5 RAW
The initial effective setting was therefore as follows:
LAW C
CW1 CW2
$\underset{R}{\mathrm{CW}}$
CW4 CW
$\underset{N}{\text { RAW }}$

One letter was enciphered, whereupon (RAVV being at $N$ for the first letter) LAW and CW were advanced one step to the following position:
LAW CW
B
H
OW2
R
CW5
RAW
0

Then CW5 and RAW were both automatically advanced, one step per letter for 26 letters, whereupon, at the 26 th letter (the 27 th of the dispatch) the wheels were in this position:

| LaW | CW 1 | CW 2 | CW 3 | CW 4 | CW 5 | RAW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | H | $?$ | R | $?$ | B | N |

${ }^{1}$ The Hebern Company furnished the Navy Code and Signal Section with a pair of machines about a year before this office received similar ones. The wiring in the rear switching plate of the Navy machines was identical with that in the machines furnished this office although the wiring of the cipher wheels was altogether different. Apparently the manufacturers had in mind a standard wiring of the rear switching plate for all all efforts of cryptanalysts.

The next letter was enciphered in this position：

| LaW | CW1 | CW 2 | CW 3 | CW 4 | CW 5 | RAW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | I | $?$ | R | $?$ | C | 0 |

Since each series of 26 letters immediately following the advance of CW1 forms a single mixed－ alphabet substitution cipher of its own，and conversely since each such single mixed alphabet is initiated by the displacement of CW1，it is advisable for this and other reasons to be detailed later，to have the dispatches in such a form that the initial letter of each line of cryptograplic text is the first letter that was enciphered just after CW1 was displaced．Since CW1 is displaced when N of RAW is at SET，the dispatches were written down on cross－section paper so that the key letter N of RAW applies to the last letter of each line，and the key letter O of RAW applies to the first letter of each line．Thus having fixed the position of the initial letters of lines，the key letters of CW5 applicable to the columns can then be indicated by the letters of a normal alphabet beginning at the correct point，and applicable to the correct columns． For example，if the apparent key setting for CW5 is A（as in the keyword AGRAM），then the eflective key setting of CW5 for the initial letter of the dispatch is B ，and the normal alphabet is written above the dispatch so that the letter B is above that colmm in win the initial fetter of the dispatch falls．Thus，Dispatch No． 1 took the following form，in which the key setting for the initial letter in each line is shown：

DISPATCH NO． 1
Key：AGRAM（Effective key：AGRBN）
（Reading of key at the beginning of each line of text is shown at the left of each line）

忥苍汸塄总 g

BHRCO NUTXHVZSLUMLZXHXHOHYBRCL CIRCO i UFCDSUFMOVKCNKYNNGAUWYLIQ Z D J R C O j UTLWBYDGOWKHRXTCJCSVGJJFYV EKRCOK JSRCEZUQKDOYTXVTVCASNQPGEC FLRCO I ARUCWT，D DCUQ IX FLCBKDBECHXDG GMRCO m VAYEEUZHWRWVVPVDVMGENJWVUU HNRCO n ENMOQJPUMVKGWQCZWKRIIXMJAC I ORCO O LNSWEAMIAUUVWVBLEMBOSPXFRR JPRCO P S G O WC J LVMHYAJEZGFYBUDAZLOQ K Q R C O q UMTZTOVTBDKWHACHYNYOBNPIHR LRRCO 0 TKSXFGWMNLNGOHYMKHPGWIEBEL MSRCO S ABLZCJUCLJXSCUDLWUTAFIARTU


It will be noted that the initial setting for the last line of text is NTRCO．Remembering that when $N$ appears on LAW the next encipherment will advance LAW and CW3，a bar is used to separate the first letter of the line $(S)$ from the rest，to indicate that a displacement of CHI ccured ath shifts，a new olphabet in introduced all the dispatches were written out in a similar manner． and wher
Another por ispatch when written as is by a letter which corresponds to the position of CW5 in the encipherment of the dispatch，and， by a letter which corresponds to the position of CWo in the encipherment of the dispatch，and， of CW1．These designatory letters may，therefore，well serve as coordinates to indicate any letter to which reference is made in the subsequent analysis．The position occupied by a letter will be referred to as its locus，which may then be given by a capital letter，indicating the column in which the letter occurs，and a small letter，indicating the horizontal line in which it occurs． Thus，locus Gi designates that position occupied by the letter in column（i，line i；the letter accompanying this locus in the case of the foregoing dispatch is $S_{c}$ ．

49．Finding NCAL5 ${ }_{c}$ equivalents．－The first step was to determine the NCALj ${ }_{c}$ equivalents and write them down under the cipher letters，as explained in paragraph 38，section IX．Al that was necessary was a fixed alphabet corresponding to RFS and a sliding normal alphabet． J is the first cipher letter．When it was enciphered，the letter B of CW5 was at SET（this from the key setting，AGRAM）．Hence，the sliding alphabet was set so that B of NAL5 was directly above $T$ of RFS．

## 

$J$ of RFS was seen to be under $K$ of the normal alphabet．Hence，$K$ was written under $J$ ， as the $N C A L 5_{c}$ equivalent of $J$ of the cipher text．Now CTH5 was in exactly the same position for every letter of the column in which $J$ is located．Hence，the $N C A L 5_{c}$ equivalents for all the letters in that column were at once written down by referring to the fixed and sliding alpha－ bets above．Thus，$S$ ，the second letter in the column is seen to he under $R$ ；$Z$ ，the third letter， is under X ，and so on，all the way down．The second letter of the dispatch， N ，was enciphere when C of CW5 was at SET．Hence the normal alphabet strip was slid one space to the left Thus：
RFS．

## 

N is now under R ，and all the letters in the same column with N can be converted．Thus the process was continued until all the cipher letters were converted into their NCAL5．equivalents．

50．Constructing the necessary tables．－It was then necessary to construct frequency tables of the kind described in paragraph 42 ，section IX．Five tables were constructed．They are for pairs with the formulas $\theta_{1} \theta_{2}, \theta_{1} \theta_{2}, \theta_{1} \theta_{2}, \theta_{1} \theta_{2}, \theta_{1} \theta_{2}$ ；these are all given in the appendix tables 15－19）．Note should be made of one fact in connection with the construction of thiese tables．

Consider the $h$ line of NCAL5．equivalents as follows：
RHELXSEZEPRHKOHXJVLWKSQTFR

In compiling the first table, recording pairs with the formula $\theta_{1} \theta_{2}$, when $Q$, the 23d letter in the line is reached, its third interval successor is R , which terminates the line. But since each basic sequence may be regarded as being in the nature of an unbroken chain, or cycle, and since these NCAL5 $5_{\mathrm{c}}$ equivalents are merely normal alphabet expressions of the basic sequence, it is perfectly legitimate to continue the tabulation of 3d interval pairs by taking the second members of pairs $\theta_{1} \theta_{2}, \theta_{1} \theta_{2}$, and $\theta_{1} \theta_{2}$, from the beginning of the same line. Thus, T. .R, F. .H, and R. .E complete the tabulations for this line. (Loci are shown above the letters.) Hence, each line of 26 letters yields 26 observations as regards pairs separated by any constant interval, that is, with any formula whatever

$$
\text { DISPATCH NO. } 1
$$

Key: AGRAM. (Effective key: AGRBN)
raw.ol P QRSTUVWXYZABCDEFGHIJKLMN cws...C D EFGHIJKLMNOPQRSTUVWXYZAB

## 

AGRBN g $\{$

BHRCO $h\left\{\begin{array}{l}N \\ R\end{array}\right.$
(



FLRCO $1\left\{\begin{array}{lllllllllllllllllllllll}A & R & U & C & W & L & D & D \\ Z & Y & U & Q & D & X & F & L & C & B & K & D & B & E & C & H & X & D & G \\ \hline\end{array}\right.$
GMRCO $m\left\{\begin{array}{llllllllllllllllllllll}V & A & Y & E & E & U & Z & H & W & R & W & V & V & P & V & D & V & M & G & E & N & J\end{array}\right.$
HNRCO $n\left\{\begin{array}{lllllllllllllllllllllllll}E & N & M & O & Q & J & P & U & M & V & K & G & W & Q & C & Z & W & K & R & I & I & X & M & J & A \\ F & S & J & H & Q & Q & H & N & P & W & Y & L & B & Z & I & N & F & F & P & 0 & P & D & D & I & X \\ T\end{array}\right.$

JPRCO $\mathrm{p}\left\{\begin{array}{lllllllllllllllllllllllll}S & G & O & W & C & J & L & V & M & H & Y & A & J & E & Z & G & F & Y & B & U & D & A & Z & L & 0 \\ S & B & G & Y & Q & C & U & F & C & N & K & X & S & M & F & A & U & I & Z & D & U & U & T & C & L\end{array}\right.$




As pointed out once before, it is really unnecessary to make tabulations of pairs with formulas greater than $\theta_{1} \theta_{2}$ because of the reversible relation existing between pairs of the formula
 same as $K{ }^{1}{ }^{12}$, etc.
51. Study of tables and reconstruction of MCAL5.-Now comes the most difficult part of the analysis-that concerned with the reconstruction of MCAL5 from the interval tables. In the table of pairs with the formula $\theta_{1} \theta_{2}$, the letter $T$ was indicated 16 times as the third interval successor of $D$ (the pair $D T=16$ occurrences), the most frequent pair in all the tables. According to the theory of solution, this meant that D. T was a sequence in MCAL5. This was assumed to be correct.
In the same table, the following possibilities for the third interval successor of T are noted (D can be omitted at once as a possibility, for a letter can appear but once in a sequence):

According to theory, T ${ }^{1}$ B, with 12 occurrences, should be correct, but T U, with 10 occur rences, runs a very close second. How can one distinguish between them or in fact, between T B, T U, T W, and TY? In such a relatively small amount of text a difference of three or four ccurrences may not be significant

Consider the relationship existing between the $\theta_{1} \theta_{2}$ table and the $\theta_{1} \theta_{2}$ table. Assuming both $D T$ and $T B$ to be correct, then the $\theta_{1} \theta_{2}$ table should show $D B$ as highest in frequency. If $T U$ is correct, then DU should be highest in frequency. But upon reference to the table it will be seen that neither D B nor D U is of greatest frequency, for the pair D 0 occurs 15 times. Which is the most probable sequence, D B B D U , or D $\mathrm{D}^{7}$ ?

Now there is no reason why the data of two or more tables cannot be combined, providing the work is done correctly. For example, if D T is correct, and if T B is correct, then D B must be correct, and the sum total obtained by adding their respective frequencies should be higher than that obtained by adding incorrect frequencies. The sum in this case is 20 , for the frequency of TB is 12 , plus that for DB, 8 , equals 20 . The following sums are noted:

$$
\begin{array}{llll}
1 & 4 \\
D T A=T & 1 \\
D & (6)+D & 1 \\
D T & \text { P }
\end{array}
$$

According to these summations, the evidence seems to be in favor of $\mathrm{DT}^{14} \mathrm{~T}^{7}$, for it has a cumulative value of 22 occurr ences, as against 20 for DTB, and only 11 for DTU. It is to be recognized,
of course, that a difference of only two or three units of frequency may not be significant at all, and such a contingency must continually be borne in mind throughout this work. But since some starting point must be established, in order that the process may be continued, the sequence D T 0 will tentatively be assumed to be correct

What about the sequence D T O 日? For this determination, three corroborative sources of
 table to see what are the likely candidates for the position $\theta \theta$. Even considering only those whose frequencies are five or orer, there are 12 candidates: B, C, F, H, J, K, L, Q, U, V, X, and Y (It is to be noted that as a letter becomes firmly fixed in its position in the MCAL5 sequence it can automatically be eliminated as a possible candidate for any other position. Thus, D, 0 , and $T$ having tentatively been fixed into position, they may be eliminated as candidates for any other positions in the succeeding calculations. It is for this reason that $0 D$ (frequency 6) is eliminated as a candidate in the attempt to continue the D T 0 sequence.)

The calculations are as follows

$$
\begin{aligned}
& D \mathrm{~T} 0 \mathrm{C}=0 \mathrm{C}(8)+\mathrm{TC}(8)+\mathrm{DC}(3)=19 \\
& D T 0 F=0 F(9)+T F(6)+D F(6)=21 \\
& D \mathrm{TOH}=0 \mathrm{H}(5)+\mathrm{TH}(5)+\mathrm{DH}(4)=14 \\
& \text { DTOJ }=0 \mathrm{~J}(5)+T \mathrm{~J}(5)+\mathrm{D} \mathrm{~J}(2)=12 \\
& \begin{array}{lll}
D T 0 K=0 K(5)+T K(5)+D K(2) & =12 \\
D T 0 L=0 L(5)+T L & (3)+D L & (7)=15
\end{array} \\
& D T 0 L=0 L(5)+T L(3)+D L(7)=15 \\
& D T O Q=0 Q(11)+T Q(6)+D Q(7)=24 \\
& D T O V=0 V(6)+T V(5)+D V(2)=13 \\
& D \mathrm{O}=0 \mathrm{X}(6)+\mathrm{T} \quad(5)+D \mathrm{X}(5)=16 \\
& \begin{array}{l}
D T 0 X=0 X(6)+T X(5)+D X(5)=16 \\
D T O Y=0 Y(8)+T Y(11)+D Y(11)=30
\end{array}
\end{aligned}
$$

Very clearly, the sequence is indicated as D T $\mathrm{T}_{0}^{1} \mathrm{O}_{\mathrm{Y}}^{\mathrm{F}}$. For the next position, there are 14 candidates, according to the $\theta_{1} \theta_{2}$ table. They are A, B, G, H, I, K, L, M, N, P, $\underset{1}{2}, R, V$, and $W$; four corroborative sources of data are available, viz, the $\theta_{1} \theta_{2}, \theta_{1} \theta_{2}, \theta_{1} \theta_{2}$, and $\theta_{1} \theta_{2}$ tables. The calculations are as follows

| 1 | 7 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $D$ | 13 |  |

Again the evidence is very clear. The sequence is D | 1 | 4 | 7 | 7 | 10 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- |

The calculations for the succeeding placements were made in the same manner, and in the majority of cases the evidence in favor of each placement was very clear-cut. Only in two or three cases was there doubt, and these were determined by special methods which suggested themselves in each case. Suffice it to say that the entire MCAL5 was reconstructed from the ten test messages, and was found to be as follows:

## 

What the initial letter of the sequence is, in other words, which letter should be placed under A of the normal alphabet in order to give a complete CAL5 is not indicated by the sequence itself. But it has been found that it makes no difference with what letter the sequence begins, for it expresses only a relative relationship between the letters composing it. Hence, CAL5 may be written as follows:

## 

In fact, the mixed sequence may be set under the normal alphabet at any one of the 26 points of coincidence, with similar results so far as encipherment or decipherment is concerned. The reader may prove this to his own satisfaction by trying out two alphabets based upon the same sequence but begimning at differeit point.
52. Reconstructing the table of basic cipher-text sequences.-Having at last reconstructed the sequence of MCAL5, the next step was to proceed at once to the reconstruction of the table of basic cipher-text sequences. It has been stated that for this purpose only a knowledge of RFS and MCAL5 is necessary. Following the procedure outlined in section VII, the first sequence of the table was obtained by setting CAL5 above the RFS so that A of NCAL5 was opposite T, the first letter of RFS. Thus:

NCAL5-. ABCDEFGHIJKLMNOPQRSTUVWXYZABC
NCAL5.- DPGTBZOHRYMSLAJIWCKUQFNVXEDPG IFS - T Y O E UMXDFJQVKWBNSHCILRZAGP

Assuming that the current enters CW5 from the first contact of BS5 (A of NCAL5), the cipher equivalent would be $E_{c}$, since the $A$ current is converted into a $D$ current, and $D$ is then pite EFS ,

Sliding CAL5 one space to the left, and still assuming that the current enters $\mathrm{CW}_{5}$ from the first contact of BS5 (now B of NCAL5), the cipher equivalent would be $B_{c}$, as shown below.
NCAL5.. A B E D F GHIJKLMNOP1
NCALín. D $\qquad$ TYOEUMXDFJQVKWBJNSHCILRZAGP

This gives the pair of letters E B as the beginning of the first basic sequence. Continuation of the process results in establishing the following sequence:

Basic sequence 1...- E B U S A L F T J N O D P W R I X V C Y Z Q H G P M

It is to be noted that in establishing this sequence the current is always assumed to enter CW5 from the first contact of BS5. It would of course be possible to reconstruct all the sequences by the same process, but there is a much shorter method.

Having reconstructed one of the basic cipher-text sequences, and having already at hand RFS, the reconstruction of the entire table followed very speedily, according to the procedure detailed in paragraph 32, section VII. The entire table is as follows

## TABLE 9

> ABCDEFGHIJKLMNOPQRSTUVWXYZ
> EBUSALFTJNODPWRIXVCYZQHGPM NMHGRJYQSEFTBZLDKI OAVCPTXU XCPZQOVHUJYNARFWLEGKITYDMS
PWMBLDKUCTGVSAXYNZEUQFIJR
BXNRFWMIYPKHGDOSAUMVJLQ ET
DSZJBXLOTVPFEHGMXKQRVAUYN
HAQNDREYBITJUCPXDWVZKGMOSF
GVSFZU NLY QMITDFBKAWPXEHC
KHJAMESROVXLYF JNWGBTDUCQIP
C Q GXUHZEKDROJQSBPNYFMIVLTW
V P DMCAUWFZEQVHNTSOJXLKRYBI
TFXIGMBJAUVKCSYHEQDRWZONLK
J D L P XNQ GMKWIHOCUVFZBAESRWY
FRTDSVPXWBLCEIMKJANGUHZBOQ
ZYFHKTDBNRIULYWQGSPMCANEVJ
O J CWY F N S Z L M R D B V P H TXIGSUKQA
Q I B O J SHARXZFNKTCYDLPHMWVGE
LNEQHCGZDAJSWYIOFRTCXBKPUV
SUVCIPAFGQHBOLEJZYIDNWTMKR
MKILTGJPVCNERUQAOLFSBYXWZH
WLRYPQTKISUZMVGERJHNODBACX
RZOTVYWLHMAXKPUZQCSEFNGIDB
AE YK O B R C X G D W TMAVIHUJSPLFNZ
U O WENZIDPFBYXGKLCMQHTRJSAG

The specific use to which this table was put will be explained later, but the general use may here be indicated. Suppose that after MCAL5 has been reconstructed from the data afforded by a few dispatches in which the $\mathrm{NCAL}_{\mathrm{c}}$ equivalents had to be determined, a few more dispatches are intercepted. It is obvious that it will be unnecessary to set down the NCAL5 ${ }_{\mathrm{c}}$ equivalents of the text of the new messages in order to establish repetitions in lines. The basic cipher-text sequences can be used directly on the lines of cipher text themselves, and thus, repetitions can be determined
53. Solution of the first line of cipher text.-All of the analysis accomplished thus far ha or its purpose the ultimate reduction of the individual lines of cipher text into single-mixed alphabet substitution ciphers. This purpose is now to be achieved by the application of the ppropriate basic cipher-text sequences to the cryptograms.

The first step is to assign, to the letters of each line of text, numbers indicating the basic equences to which they belong. This will show what repetitions occur within each line. The process when applied to Dispatch No. 1, for example, yields the following:

Dispatch No. 1
Key: AGRAM1 (Effective key: AGRBN)

```
CW5.....- CDEFGHIJKLMNOPQRSTUVWXYZA
謟隠
BHRCO_(..._
```



The entire dispatch was then treated in the same manner. Referring now to the first complete line of text, after about 45 minutes experiment, the following decipherment was obtained:


```
residentofth e (\frac{n}{(})
```

Note that the encipherer made an error in regard to the $U$ of United. The analysis showed that the cipher letter X was to be assigned the number 18 , which is the same as that for the next letter H . But the decipherment shows that the letter X should not belong to the same basic sequence as does the letter $H$, for $X_{c}=U_{p}$ and $H_{c}=N_{p}$. However, the presence of this rror did not retard the decipherment.

Now it is obvious that the deciphered clear text of each line will suggest assumptions for deciphering the next line, with the aid given by the indicated repetitions. In this case it was somewhat unfortunate that only one letter of the next word was given, $I$, but it nevertheles .in ath . IS


## FURTHER STEPS IN ANALYSIS


54. Reconstruction of Alphabet 1.-In each column of cipher text, when the dispatch is arranged in horizontal lines of 26 letters each, the cipher letters represent encipherments at exactly the same position of CW5. In successive horizontal lines, only CW1 has undergone displacement one step, providing the "liey" has not been such as to displace CW3 also. I Dispatch No. 1, CW3 remains stationary until the very last line of text, hence only the successive displacements of CWI need be considered in studying the successive limes up to the last one Now note that in column C of Dispatch No. 1, for cxample, the initial letter of the $i$ and $j$ fines is $U_{c}$ in both cases. Decipherment showed that in the first case, $U_{c}=N_{p}$; in the second $\mathrm{U}_{\mathrm{c}}=\mathrm{C}_{\mathrm{D}}$. Only a single displacement of CW 1 has brought this about, for $\mathrm{CW} 2,3,4$, and 5 are in exactly similar positions in the two cases. This means, in other words, that the current originated by N in the first case enters the fixed contact in BS2 at exactly the same point as the current originated by C in the second case, for both of them must of necessity have traversed exactly the same path through CW2, 3, 4, and 5 in order to produce $U_{c}$ at RFS. The two positions of NAL1 may be diagrammatically illustrated as follows:


In the first case $N_{D}$ is over $Q$ of NAL1; in the second case $C_{D}$ is over $T$ of NAL1. For convenience in reference, letters found in this way will be designated hereafter as the plain-text NAL equivalents. Between the two cases CW1 has advanced one interval. Hence $T$ follows Q in IAL1, that is, QT forms a pair of sequent letters in MAL1. Searching for other cases of a mirar nature, in loci Ki and Kj it is found $N$ c occurs in the $\mathrm{C}_{\text {a }}$. OAL1 will show that this, however is but a corroboration of the first case above described, howing the seguence QT to in ML1. In loci Mi and Mj the letter K oceus in the first instance representing 0 , in the second $\mathrm{E}_{\mathrm{p}}$. Referring to the diagram of alphabets impedi ity $\mathrm{E}_{\mathrm{p}}$ is B in the second instance. Hence WB forms a sequence in MAL1
In loci $\mathrm{Ph}, \mathrm{Pj}$, and Pk , the letter $\mathrm{X}_{\mathrm{c}}$ occurs. Attention was called to the fact that the first-named X is an error; hence, it cannot be considered in establishing values in MAL1. But
$x_{c}$ in locus Pj represents $\mathrm{O}_{\mathrm{p}}$, and in locus $\mathrm{Pk}, \mathrm{P}_{\mathrm{D}}$. The two positions of NAL1 in these cases are as follows:

LFS


In the first case the $N A L 1_{\mathrm{p}}$ equivalent of $\mathrm{O}_{\mathrm{p}}$ is X ; in the second case, the $\mathrm{NAL} 1_{\mathrm{p}}$ equivalent of $P_{D}$ is $E$. It therefore follows that XE forms in sequence in MAL1. The following additional sequences in MAL1 are established by analyzing these five lines of deciphered text of Dispatch No. 1:

$$
\begin{aligned}
& \mathrm{C}_{\mathrm{c}}\left\{\begin{array}{l}
\text { locus } \mathrm{Fk}=\mathrm{A}_{\mathrm{p}} \\
\text { locus } \\
\mathrm{Fl}=\mathrm{E}_{\mathrm{p}}
\end{array}\right\} \text { giving sequence HD in MAL1 } \\
& \mathrm{D}_{\mathrm{c}}\left\{\begin{array}{ll}
\text { locus } & \mathrm{Ij}=\mathrm{L}_{\mathrm{p}} \\
\text { locus } & \mathrm{Il}=\mathrm{E}_{\mathrm{p}}
\end{array}\right\} \text { giving sequence Y.D in MAL1 } \\
& \mathrm{U}_{\mathrm{c}}\left\{\begin{array}{l}
\text { locus } \mathrm{Lh}=\mathrm{F}_{\mathrm{p}} \\
\text { locus } \mathrm{Ll}=\mathrm{C}_{\mathrm{D}}
\end{array}\right\} \text { giving sequence G. . .V in MAL1 } \\
& \mathrm{C}_{\mathrm{c}}\left\{\begin{array}{l}
\text { locus } \mathrm{Rj}=\mathrm{E}_{\mathrm{D}} \\
\text { locus } \mathrm{Rl}=\mathrm{R}_{\mathrm{D}}
\end{array}\right\} \text { giving sequence B.O in MAL1 } \\
& \mathrm{C}_{\mathrm{c}}\left\{\begin{array}{l}
\text { locus } \mathrm{Tj}=\mathrm{L}_{\mathrm{p}} \\
\text { locus } \mathrm{Tk}=\mathrm{A}_{\mathrm{p}}
\end{array}\right\} \text { giving sequence YH in MAL1 } \\
& \mathrm{A}_{\mathrm{c}}\left\{\begin{array}{l}
\text { locus } \mathrm{Ul}=\mathrm{P}_{\mathrm{D}} \\
\text { locus } \mathrm{Uk}=\mathrm{R}_{\mathrm{b}}
\end{array}\right\} \text { giving sequence C.N in MAL1 }
\end{aligned}
$$

Thus far the following sequences have been established:

$$
\begin{array}{lllll}
\text { QT } & \text { XE } & \text { HD } & \text { Y.D } & \text { B. } 0 \\
\text { WB } & \text { YH } & \text { C.N } & \text { G...V } &
\end{array}
$$

By virtue of the letter common to the two pairs YH and HD, they may be joined, making YHD. Confirmation is seen in the sequence Y.D, established independently. No other unions can be made.

It is obvious that if there were a sufficient number of repetitions of cipher letters in the columns of these five lines of text, the entire sequence could be established. But there is, in reality, a way of overcoming this insufficiency of repetition. The process is somewhat complicated, but useful.

Suppose we approach the problem from a somewhat novel viewpoint. Take the case of $R_{\mathrm{D}}=\mathrm{N}_{\mathrm{c}}$ in locus Ch. If there were another $\mathrm{R}_{\mathrm{D}}$ in locus Ah, what would the cipher letter be? Reference to the table of basic sequences is made. Applying that sequence in which N occupies he first position (to correspond with column A of the dispatch) it is found that $B_{c}$ would be the letter. That is, if there were an $R_{p}$ in locus $A h$, it would equal $B_{c}$.
Now if there were a $B_{c}$ in any one of the loci $\Lambda g, ~ A i, ~ A j, ~ \Lambda k$, or $\Lambda l$ (i.e., within the body of already deciphered text) its plain-text equivalent would, of course, be known. Now there is no $\mathrm{B}_{\mathrm{c}}$ in any of these loci, but if it were present, and if its plain-text equivalent were repeated plain-text letter's cipher equivalent with a letter of the basic sequence initiated by B would
occur. Therefore, let this basic sequence initiated by B be applied to the five lines of deciphered text to see if any such coincidences can be found. Thus:

|  |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |

The following coincidences are noted:

$$
\begin{array}{ll}
\mathrm{R}_{\mathrm{c}} \text { in locus Dl }\left(=\mathrm{I}_{\mathrm{v}}\right) & \mathrm{D}_{\mathrm{c}} \text { in locus N1 }\left(\mathrm{also}=\mathrm{I}_{\mathrm{b}}\right) \\
\mathrm{W}_{\mathrm{c}} \text { in locus } \mathrm{Fj}\left(=\mathrm{N}_{\mathrm{p}}\right) & \mathrm{K}_{\mathrm{c}} \text { in locus Kk }\left(=\mathrm{H}_{\mathrm{p}}\right)
\end{array}
$$

That is, $B_{c}$ equals, successively in loci $A h, A j, A k$, and $A l$, the letters $R_{p}, N_{p}, H_{p}$, and $I_{p}$.
Determining the plain-text normal-alphabet converted equivalents, as before, for the sucessive displacements of AL1, the sequence K.RVJ is obtained

Take the second letter of line $h$, viz, $\mathrm{U}_{\mathrm{c}}=\mathrm{E}_{\mathrm{p}}$. If $\mathrm{E}_{\mathrm{p}}$ occurred in locus Ah , its equivalent ould be $Y$. Applying the basic sequence in which $Y$ is the initial letter to the lines of deciphered text, the following coincidences are found:

$$
\mathrm{Q}_{\mathrm{c}} \text { in locus } \mathrm{Ml}\left(=\mathrm{T}_{\mathrm{D}}\right) \text { and } \mathrm{T}_{\mathrm{c}} \text { in locus } \mathrm{Qj}\left(=\mathrm{M}_{\mathrm{p}}\right)
$$

That is, $Y_{c}$ equals successively in loci $A h, A j$, and $A l$, the letters $E_{D}, M_{p}$, and $T_{D}$.
Determining the plain-text normal-alphabet converted equivalents, as before, for the suc essive displacements of AL1, the sequence Z.V.Q is obtained

When all of the text is analyzed in the same manner, the results shown in the table below re found. In this table the top line gives the cipher letters that would result if the plain-tex letters under the corresponding cipher letters occurred in locı $\mathrm{A} q, \mathrm{Ah}_{\mathrm{h}}, \mathrm{Ai}, \mathrm{Aj}, \mathrm{Ak}$, and Al

$$
\text { TABLE } 10
$$

| A B C D F G H I J K M N O P Q R T UVWX Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| g |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h |  | R | I |  | 0 |  |  | A |  |  | S |  |  | D |  |  |  |  |  | E | F |
| i |  | U |  | N | S | V | D | P | A |  |  |  | T | C | 0 |  |  | I |  |  |  |
| j | I | N | K | C |  | B | 0 |  |  | L |  |  | A |  | E |  |  | S |  | M |  |
| k |  | H |  | 0 |  |  | P | R | T | A | J | E |  | S |  |  | C |  |  |  | D |
| 1 |  |  | U |  |  | N |  |  | M | E |  |  | 0 |  | R |  |  |  |  | T |  |

Determining the plain-text normal-alphabet converted equivalents for each placement of NAL1, the following sequences are established:

|  | Sequence |  | Sequence |
| :---: | :---: | :---: | :---: |
| For $\mathrm{B}_{\mathrm{c}}(\mathrm{R} . \mathrm{NHI})$ | K.RVJ | For $\mathrm{O}_{\mathrm{c}}$ (PN. . E ) | AP. . C |
| For $\mathrm{C}_{\mathrm{c}}(\mathrm{U} . . \mathrm{D})$ | B. . S | For $Q_{c}$ (DC.S $)$ | 0S.L |
| For $\mathrm{D}_{\mathrm{c}}(\mathrm{I} . \mathrm{K} . \mathrm{U})$ | F.P.E | For $\mathrm{Rc}_{\mathrm{c}}(\mathrm{OE.R})$ | WB. 0 |
| For $\mathrm{E}_{\mathrm{c}}(\mathrm{NCO})$ | QTY | For $\mathrm{S}_{\mathrm{c}}(\mathrm{H}$. . W) | S. F F |
| For $\mathrm{F}_{\mathrm{c}}(0, \mathrm{~S})$ | VJ | For $\mathrm{T}_{\mathrm{c}}(\mathrm{FP})$ | HD |
| For $\mathrm{G}_{\mathrm{c}}^{\mathrm{c}}$ (VB.N) | VJ.T | For $\mathrm{V}_{\mathrm{c}}^{\mathrm{c}}$ (IS) | GK |
| For $\mathrm{H}_{\mathrm{c}}$ (DOP) | : PXE | For $\mathrm{X}_{\mathrm{c}}$ ( NA ) |  |
| For $J_{c}($ AP, R $)$ | EC.N | For $\mathrm{Y}_{\mathrm{c}}($ E.M.T) | Z.V.Q |
| For $\mathrm{K}_{\mathrm{c}}(\mathrm{A} . \mathrm{TM}$ ) | F.PX | For $\mathrm{Z}_{\mathrm{c}}(\mathrm{F} . . \mathrm{DC})$ : | G. .RV |
| For $\mathrm{L}_{\mathrm{c}}$ (LAE) | YHD |  |  |

Assembling and joining sequences, the following result is obtained:
EC. NGKZRVJQTYHDWB. OS I LFAPX

The entire sequence of MAL1, with the exception of two letters, las been reconstructed The two missing letters are $M$ and $U$, which must be inserted between $B$ and 0 , and $C$ and $N$ Only two possibilities exist, BMO or BUO, and CUN or CMN. The exact placements were easily found later by trial on text, and the completed secuuence was established as follows:

ECUNGKZRVJQTYHDWBMOSILFAPX
This sequence is, of course, the converted equivalent of the real MAL1, and in order to produce the latter it is only necessary to make use of the enciphering-deciphering relationship existing between such alphabets. By setting the sequence under the normal alphabet (the first alphabet below) and finding reciprocal equivalents, the second alphabet is obtained:
$\begin{array}{llllllllllllllllllllllll}A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T & U & V & W & X \\ E & Y & Z \\ C & U & N & G & K & C & \text { Converted equivalent }\end{array}$


This equivalent of the real MAL1 (it is only an equivalent because it may not coincide letter-for-letter with the real M1AL1, though it will work just as well) may be used for enciphering or deciphering by means of the sliding strips.
55. Using the reconstructed Alphabet 1 as ant aid to further decipherment. Once Alphabet 1 has been reconstructed in the manner described above, from a few lines of deciphered text, this alphabet may be employed to aid in the further decipherment of the dispatches. To illustrate the method, consider the pline of Dispatch No. 1, in which, as shown by the numbers beneath, the 8 th, 11 th, 13 th, 16 th, and 20 th letters represent the same plain-text letter. One of the cipher
equivalents, U , in locus V , also appears in the same column, line i , where it represents $\mathrm{A}_{\mathrm{p}}$. The diagrams of the position of Alphabet 1 for the two cases are as follows:

|  |  |
| :---: | :---: |
|  | B SXRZTKDNGCHMVO |
|  |  |

In the first case, $A_{p}$ in the LFS is over F of NAL1 which is converted into W of MLAL1, and $W$ is then opposite the fifteenth fixed contact of 13 S2. Since the cipher letter $U$ is the same for both cases, it means that in the second case the current must cnter CW2 at exactly the same point as it does in the first case. This will be at D of NALA, which is the conversion the same point of NALI, and the latter is under I of LIFS. The letter I is, therefore, the plain-textletter that $\mathrm{U}_{\mathrm{c}}$ represents in the second case. Hence, the 8 th, 11 th, 13 th, 16 th , and 20 th letters in line p all represent $I_{p}$. No assumptions based upon frequency were necessary the results being positive and definite when Alphabet 1 has been reconstructed. The relations between all similar letters in columns can likewise be established. Thus, decipherment is considerably facilitated.
56. Reconstructing a single-alphabet equivalent of Alphabets 2,3 , and 4 .- It will now be shown how the subsequent decipherment of the message can be still more facilitated by reconstructing a single alphabet which will serve to give all the final results that the combined effect of the interaction of $\mathrm{Alphab}_{\text {pets }} 2,3$, and 4 produces. It will be obvious that providing no displacement of CW2, 3, or 4 occurs during the encipherment of a message a current entering any given LHC of CW 2 will always emerge from the same RHC of CW4. Hence, so far as the results in the case of each single message are concerned, Alphabets 2, 3, and 4 act as a single unit. Now consider the chinerment of amessage of say 300 letters, during which CW 2 , 3 , and 4 undergo no displacerne equivalents of the same letter in each column will be due should be posible to
 application, being applicable only to that then ouly tu that applitan CiF , 3 , or 4 occurs. Within these limits, howe it will or pherment.

The process of reconstruction is as follows:
Set Alphabets 1 and 5 in positions to correspond to their respective positions when the letter $\mathrm{N}_{\mathrm{c}}$ in locus $\mathrm{Cl}_{1}$ of Dispatch No. 1 was enciphered, and between them place the normal alphabet set at the proper letter so far as the setting of CW3 was concerned. Thus:

## LFS.

AL1
Equivalent
of AL2-3-4
of AL2-3-4-
AL5.........


The case under discussion is that where $R_{v}$ equals $N_{c}$. R in LFS is over $K$ of NAL1, which is converted into $F$ of MAL1, but $F$ of NAL1 is then opposite $P$ of the normal component of the single-aphahet equialent or Ac-3-4, hereater abbrevated NE-3-4. Now N of is under $R$. NALs and $R$ was produced by conversion of 1 of ALs into $R$ of MaL5. It It follows, therefore that P enter NAL5 at 1 , the current had to emerge from X of NEAL2-3-4 under P in NEAL2 $-3-4$. Thus the position of a letter of MEAL2-3-4 has been determined. By tracing other letters through in the same manner, using only the first two lines of deciphered materiall, the following additional placements in MEAL2-3-4 are determined:

This reconstructed MEAL2-3-4 will very greatly facilitate the decipherment of all the succeeding lines up to that in which CW3 has advanced, whereupon a new MEAL2-3-4 becomes effective.
7. Application of foregoing principles to another dispatch.-Having shown ho Heconstructed may be reconstructed form but two lines of dow be demonstrated using Dis patch No. 2. The first two lines, deciphered by the application of basic principles (indication of repetitions and solution of single-mixed alphabet lines) are as follows:

## key: CObaN

 (zepplincompanyacoeptallgen-Plain


From these two lines the following MEAL2-3-4 can be reconstructed

Applying this reconstructed alphabet, in conjunction with the other alphabets, to the next line of cipher text, the following decipherment is obtained:

The missing letters are easily inserted by context, and at the same time the letters lacking in previously incomplete MEAL2-3-4 can be inserted.
It becomes apparent, therefore, that when only a very few lines, say 4 or 5 , of one message have been deciphered by basic processes, the decipherment of all the rest of the messages may be attained almost directly, as a result of the reconstruction of MCAL5, MCAL1, and MEAL2-3-4.

## Section Xil

SOLUTION WITHOUT PRELIMINARY analysis of any line of text
$\qquad$ Grouping the letters of the text into o-...............................

Identifying the values of members in the same | Par. |  |
| ---: | ---: |
| 58 |  |
| 59 | Tl | The initial determination of the value of $a$ member of any category Constructing a table of basie plain-text sequences 61

62
58. Introductory statement.-In the preceding paragraphs it was shown how the decipher ment of a message could be facilitated aiter only 2 or 3 lines of text of the dispatch had been solved more or less laboriously by first principles. It will now be shown how a message can be solved more or less laboriously by first principles. It will now be shown how a message can be solved that it has been considered necessary to devote a separate section to its explanation, although it is not very complex. In brief, the procedure is as follows: The letters of the message are all distributed or grouped into but 26 classes or categories corresponding to the 26 basic sequences By virtue of a relationship existing between basic cipher-text sequences, to be explained, the soluion of a single letter in each category yields the equivalents of all other letters in that caterory and the solution of but three or four categorics yields solution of all the other categorice.
. Grouping the letters of the text into categories. - When the table of basic cipher-text sequences has been completely reconstructed it becomes possible to determine the position ocelupied in the table by every cipher letter of the cryptographic text of a dispatch. Now since there are but 26 basic cipher-text sequences in the table, it follows that ail of the elements of the cryptographic text can be allocated to but 26 different classes or categories. Thus, if the succes sive sequences are numbered arbitrarily from 1 to 26 , then a certain number of cipher letters wil all into class 1, another number into class 2, and so on. For example, taking the first column o the cipher-text of Dispatch No. 1, reading NUUJAVELSUTAS, and referring to column C of table , the reconstructed table of
 and 1 the the the $T$ in the fourthe and so on. Te these bacicsequence numbers be in the tenth, the lete he the with respect to the rest of the columns of the cipher-text will result in assigning a number to each letter of thic text, and these numbers will correspond to the particular hasic cipher-tevt sequence in which each letter in the cipher-text belongs. Thus, there will finally result a dis tribution of the elements of the text into the 26 different classes or categories mentioned above.
60. Identifying the values of members in the same category.-Now it has been shown above hat through a knowledge of MCAL1, a relation of such a nature exists between similar cipher letters in the same column of cipher text that if the plain-text equivalent of one of the cipher etters is known, those for all the other similar cipher letters in that column can be derived. Take the three U's in column C of Dispatch No. 1, for example. If the value of the first U, locus Ci
$\qquad$
is known to be $N_{D}$, then the value of the second $U$, locus Cj , can be found through the interme diacy of Alphabet 1 , to be $C_{p}$, and that of the third $U$, locus $C q, I_{p}$. Now, ha ring thus determined that $\mathrm{U}_{\mathrm{c}}=\mathrm{C}_{\mathrm{p}}$ in locus Cj the value of $\mathrm{O}_{\mathrm{c}}$ in locus Kj and of $\mathrm{R}_{\mathrm{c}}$ in locus Oj , in the same horizontal line, must also be $C_{p}$, because these cipher letters coincide with the letters of the basic cipher-text sequence in which , the first etter of that line, is found (by apphication of the basic cipher-text sequence to that line of cipher text. Again, in the case of the thir $\mathbf{U}$, which is in ocus Cq ,
 with $U$, ms lo $I_{p}$. By virtue of the forecoing process the values of all those letters of each line which belong to the same basic cipher ant sequence could readily be derived. But such on assumption the nately carries us little further. Howerer does it really male any difference whether all the nately carries us little further. However, does it really make any difference whether all the lines begin with $U_{c}$ ? Is it not still true that those letters of the line which do belong to the
same basic alphabet cipher-text sequence as does the $U$ have the same value that the $U_{c}$ would have, if it were present? For eximple, if the letter in locus Ck were also a $U$ (instead of a $\mathrm{J}_{\mathrm{c}}$ ), by reference to Alphabet 1 , its value would be found to be $0_{p}$. Now set that basic cipher-text sequence in which $U$ orcupies the third position against the line of upper cipher text and observe what coincidences are present. Thus:

$$
\begin{aligned}
& \text { JSRCEZUQKDOYTXVTVCASNQPGEC } \\
& \text { USALFTJNODPWRIXVCYZQHGPMEB }
\end{aligned}
$$

Note the four coincidences. Does it not follow that the D, S, P, and E of the line of cipher-text must all equal $O_{w}$, even though $U_{c}$ does not begin that line of text? And does it not follow that the value of any $S_{c}$ in column $D$, of any $A_{c}$ in column $E$, of any $L_{c}$ in column $F$, and so on can be derived in the same manner, by finding what $U_{c}$ would equal if it were present as the first letter thus made possible through the identification of but one letter, $\mathrm{U}_{\mathrm{c}}$ in locus Ci

A thorough comprehension of this principle will show that if the value of a sibgle member of one category of leters (those that belong to the same basie cipher-text sequence) no matter in what horizontal ine of cipher test that single member oceurs, can be correctly determined, the value of all other members of the same category can be derived through the reationship herewith disclosed. It also follows that the correct determination of the values of but 3 or 4 members of different categories will soon result in producing combinations of high degrec of probability (syllables and the skeletons of words of plain text), which will soon lead to a complete resolution of the text to be deciphered.
61. The initial determination of the value of a member of any category.--- But the question is this: How can one determine the initial 3 or 4 correct values upon which all this depends? The answer is not difient to find. After the etters have anl been distrinuted into their respec-
 istelicinter in the following the value of of the mer of the

 $S, D$ L) the plain-text ralucs derived from the isitial ascumption should form a aod asportment of these high-frequency letters. If, therefore, the derived values, based upon the assumption for $\mathrm{E}_{\mathbf{p}}$ gives a good assortment of high-frequency letters, then the assumption is likely to be correct. If the basic assumption does not yield a good assortment of high-frequency letters, then another basic assumption is made, and its derived assortment determined. It is clear that
the assumption which yields the best assortment of high-frequency letters is most likely to be the correct one.

Now it will, of course, be a great advantage to be able quiclily to determine the values derived from a number of assumptions. Heretofore, it has been necessary to refer to the sliding strips for each derivation, and the process takes considerable time. If a more direct method can be devised it would, of course, greatly facilitate the process of analysis. The method which was devised is described below
62. Constructing a table of basic plain-text sequences.-It was shown in paragraph 60 how the plain-text equiralents of similar cipher letters in columns could be found through the intermediacy of Aphabet 1 and LFS. It is obvious that the plain-text equivalents for simila cipher letters in columns can be determined once and the case. It will be recalled that provid It may be adisable are the encipherment of the same dispatch, then ing no displacement of equivalents for two or more identical plain-text letters in the same her ine solely to the displacement of CW5. On the other hand, under the same circum-
 equivalents for two or more identical cipher letters in the same column is due solely to the dis placement of CW1. If, in the latter case, Alphabet 1 and LFS are both known, then the plaintext equivalents for suci identical cipher letters can easily be determined. Looked at from the point of view of decipherment, there are only 26 contact points through which current emerges from BS2 and passes into the RHC's of CW1, and if Alphabet 1 and LFS are known then it follows that a fable of basic serfuences for plain-text letters, similar in its nature to that for cipher-text letters can be constructed.
For illustration, suppose Alphabet 1 is in its initial position, with A of its normal component opposite SET. Thus

LFS.

AL1.


AL2.
In this case, suppose in encipherment, a current enters the A of NAL2. What is the plaintext letter involved? Tracing it backward, it will be found that $\mathrm{Z}_{\mathrm{p}}$ is the letter involved. Now advance Alphabet 1 to the next position, with B at SET, and see what plain-text letter wil plain-text is as follows:
ZSECXTUCVBKNMPLDHWJFOYGRQI

That is, for example, when CW 1 is set at R , then $\mathrm{W}_{\mathrm{D}}$ will bring the current to A of NAL2.
Now there are, of course, 26 points to which a current can be brought to the letters o NAL2 from the letters of MAL1, and for each of these points a different series of plain-text letters will apply. All the sequences may be determined in exactly the same manner as that illustrated above for the first sequence. They are given below
table 11.-Table of basic plain-text sequences

## BCDEFGHIJKLMNOPQRSTUVWXYZ

 $\left.\begin{array}{lllllllllllllllllllllll}\text { Z X P } & V & K & C & F & Q & W & G & Y & U & I & D & R & J & S & M & O & E & N & H & T & B & L\end{array}\right)$ EHZNALUDOQJTSFCMYKGRIVWXB CRDJOEKVYWZBUIGHLTNXAMPSFQ XKWVQTMLPRFEANCOZDSJHUBIYG TPMYZHOUXIQJDGVRKBWCEFALNS UHLRCVESAYWKNMXTFPGQIJODBZ COXGMQBJLPTDHSZIUNYAWVKFRE BDCLIP NVOA M Q
 PYG NVIBRPAMSGYOKFEJHRDWXCU DMFRUJHBNLOT W CXKPSZGE HJIXEWLCFDOMZAYPGSTUBRNQVK WASQPOGTKVHRJLUNBZEFXDYMTC JBYUVNATMCXWOEDFRQISKLHZGP FLEMDJZHGSPVQKIXYABTOCRNUW
 Y C TPXGDFEHLZJBOWIRMNSKQUAV GZUSNKIQCORWFVPAXHDBTYEJML REBDTAYGVXPIMUJSCKFZLQWHON Q FKZJLNMSUAHEWBGTIROYPCVDX I TRWODHBEJCQPFNZAXVLUGMKSY

Now it is obvious that the ZSEC sequence, for example, does not necessarily apply only to those cases in which the current is brought to A of NAL2. Exactly to which contact point the current is brought depends upon the position of CW2. But the point is that no matter to what letter of NALL2 the current is brought for any position of CW 2 , the sequence of plain-text letters ZSE... will aluays bing the current to the same tetter of NiLa, and, proriainy 110 displacement of CIV, 3 , or 4 takes place, the cipher resultant will atways be the saine for the column to which that
 column for the $i$, to all the other seguences of table 11

Now recall what has been said about the 26 categories of letters discussed under paragraph 59. Any one of these categories may apply to any one of the sequences in table 11. But the correct assumption for the value of one member of one category will give the values for all other members of the same category. For cxample, suppose a category bearing the number 6 is being examined, and suppose that in that line which corresponds to the placement of $L$ of bearing the number 6 in a line corresponding to the placement of $M$ of CW 1 at SE' $\Gamma$ will have the value $\mathrm{C}_{\mathrm{D}}$; the value of any letter bearing the number 6 in a line corresponding to the placement
of N of CW 1 at SET will have the value $\mathrm{O}_{\text {, }}$, and so on, because in table 11, the column in which $E$ is situated in the L line read ECOIFXQ.
Perhaps the application of these principles to an actual example will best serve to clarify the process described
63. Application of principles to an actual example.-For this purpose, Dispatch No. 3 of the series submitted for test will be employed

First, all the letters of the text were assigned numbers applicable to the categories to which they belong. This was done, of cou

$$
\text { DISPatch No. } 3
$$

Key: BLOIS



CMOEO J J N N I I R R N N L I I











It will be noted that only the first letter of the last line is assigned a number, because with the encipherment of the next letter, CW3 has advanced one space. (Refer to the key for the dispatch.) Only letters belonging to the text enciphered during the nondisplacement of CW 2 , 3 , or 4 can be distributed into the same set of categories.

Then a frequency table is compiled in order to find those categories which have the most representatives. In addition to this feature of the work it is well to prepare an index giving the exact locus of each member of each category, so that it can quickly be found when necessary below
( UFKUYAMZRAFUSDW mnnnnnpprrsstuv
$2\left\{\begin{array}{l}\text { RENEZ } \\ \text { qtuww }\end{array}\right.$
3 PXSVDNUX
$3\left\{\begin{array}{l}\text { PXSVDNUX } \\ \text { lopppsuu }\end{array}\right.$
4 \{ XPIQCRKOSKC
lmttuvvvw
$5\left\{\begin{array}{l}\text { OQSAMTJEVF } \\ \text { l111notvvw }\end{array}\right.$
$6\{$ VJRXCFCOYMIMT
\{ lnnnnosttuvw
$7\left\{\begin{array}{l}\text { JBOQTNQNSCMWOQF } \\ \text { llmmmnooottuuv }\end{array}\right.$
8 \{ HJOOYRXZT
\{ ppprrsttv
$9\{$ ZQUBWGJLPQH
ZKEYKKRUCHU
10 ZKEYKKRUCHU
11 | HSVLNNWYBHIJNPX
11 mmmnpqqqqrrsww
$12\{$ AEOAUMCWN
$13\left\{\begin{array}{l}\text { IRTAFQUQUIKOTBZBV } \\ \text { opppgqquIsssssssu }\end{array}\right.$

TABLE 12
$14\left\{\begin{array}{l}\text { PGEALRXZGRSY }\end{array}\right.$ npqqvvvvwww
$15\left\{\begin{array}{l}\text { NFSJMEFTWE } \\ \text { Imnosuuuux }\end{array}\right.$
16 GITBGTXHKDILOB
$6\left\{\begin{array}{l}\text { Gnnirsuavwww } \\ \text { nnnnorsuan }\end{array}\right.$
( RTKKLPISLTVABGaD Rmmpppaqttttuvvw
18 \{JEOLXSWZLQW $\left\{\begin{array}{l}\text { JEOLXSWZLQW } \\ \text { mnnqqrruww }\end{array}\right.$ KCELXCICGOZFLYAGH
$19\left\{\begin{array}{l}\text { KCELMmpacziss } \\ \text { llmmmppaq }\end{array}\right.$
11mmmmppqqqrssst
$20\left\{\begin{array}{l}\text { RBDPYZPY } \\ \text { mmnoootv }\end{array}\right.$
21 \{ YWBVMGSDJPB
21 lopqrsstvve
$22\{$ TZLRDQHCVDJPZN
(llooopqqrsuuuv
23 WDNHMFTEGISYC
umwnwyhpfanus
24 LUMWDWXNHPFAMUJ
25 HVJDEXDEBQV
25 \{ nnqqrirstv
26 MGYVPKA
Another table is now prepared, to show the prefix and suffix to every mernber of each category. This is exactly analogous to any ordinary trigraphic frequency table used in cryptanalysis. It is shown on pare 76

```
1{\begin{array}{rrrrrrrrrrrrrrrrrrrrr}{718}&{18}&{16}&{6}&{9}&{17}&{10}&{13}&{18}&{25}&{13}&{10}&{23}&{5}\\{11}&{16}&{11}&{25}&{7}&{16}&{11}&{13}&{18}&{23}&{21}&{13}&{17}&{5}&{14}\end{array})
```



```
3 {}{\begin{array}{rrrrrrrllll}{5}&{21}&{13}&{12}&{19}&{15}&{15}&{15}\\{5}&{20}&{13}&{9}&{14}&{13}&{13}&{23}
4{[\begin{array}{rrrrrrrrl}{23}&{7}&{19}&{20}&{25}&{7}&{21}&{22}&{14}\\{24}&{24}&{16}\end{array}]
4{[\begin{array}{rrrrrrrrrrrrrrrr}{23}&{7}&{19}&{20}&{25}&{7}&{21}&{22}&{14}&{24}&{16}\\{21}&{7}&{5}&{10}&{21}&{23}&{14}&{21}&{8}&{16}&{17}\end{array}]
5{[{rrrrrern
6{[\begin{array}{rrrrrrrrrrrrrrrrrrll}{24}&{16}&{7}&{24}&{16}&{12}&{13}&{12}&{8}&{18}&{9}&{16}&{14}\\{23}&{1}&{15}&{1}&{20}&{16}&{22}&{20}&{8}&{2}&{21}&{11}&{10}\end{array})
7{[rrrrracrern
8{\begin{array}{llllrrrrrrrrr}{14}&{19}&{11}&{24}&{25}&{9}&{7}&{6}&{4}\\{19}&{17}&{17}&{9}&{18}&{21}&{6}&{17}&{24}\end{array}}
9{\begin{array}{rrrrrrrrrrrrrrrr}{1}&{20}&{5}&{12}&{3}&{19}&{11}&{26}&{8}&{24}&{17}\\{1}&{22}&{26}&{7}&{24}&{11}&{26}&{21}&{13}&{8}&{6}\end{array}}
0{[\begin{array}{lllrrrrrrrrrrrrr}{26}&{15}&{22}&{24}&{25}&{5}&{4}&{17}&{21}&{14}&{6}\\{12}&{22}&{23}&{1}&{18}&{17}&{1}&{17}&{16}&{16}&{25}\end{array})
11{\begin{array}{rrrrrrrrrrrrrrrrrrrrrr}{26}&{20}&{1}&{1}&{1}&{12}&{21}&{18}&{14}&{9}&{11}&{13}&{6}&{16}&{18}\\{17}&{7}&{24}&{5}&{8}&{19}&{18}&{19}&{22}&{11}&{9}&{13}&{16}&{18}&{1}\end{array}}
12{[\begin{array}{llllllll}{10}&{20}&{7}&{20}&{13}&{18}&{23}&{13}\end{array}]
13{\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrr}{23}&{22}&{3}&{1}&{14}&{26}&{23}&{9}&{16}&{24}&{11}&{3}&{21}&{1}&{19}&{19}&{3}\\{15}&{3}&{12}&{21}&{19}&{2}&{21}&{1}&{22}&{11}&{19}&{24}&{1}&{12}&{19}&{6}&{15}\end{array})
14{[\begin{array}{lllllllllllllllll}{18}&{23}&{3}&{19}&{4}&{25}&{1}&{20}&{5}&{18}&{14}&{ll}\end{array}]
5{
5{[\begin{array}{rrrrrrrrrrrrrr}{26}&{19}&{6}&{13}&{19}&{21}&{15}&{23}&{13}&{1}\\{5}&{26}&{16}&{10}&{3}&{15}&{23}&{3}&{3}&{-}\end{array}]
16{\begin{array}{rrrrrrrllllllllllr}{11}&{25}&{15}&{1}&{6}&{18}&{12}&{23}&{22}&{10}&{10}&{4}&{11}&{26}\\{25}&{6}&{1}&{6}&{23}&{13}&{19}&{23}&{18}&{2}&{24}&{6}&{11}&{4}\end{array})
17{[\begin{array}{rrrrrrrrrrrrrrrrrrrrrlll}{5}&{11}&{18}&{8}&{17}&{8}&{22}&{2}&{10}&{1}&{10}&{8}&{24}&{7}&{14}&{4}\\{5}&{18}&{19}&{17}&{1}&{22}&{25}&{23}&{7}&{10}&{7}&{25}&{23}&{9}&{21}&{15}\end{array})
18{\begin{array}{rrrrrrrrrrrrrrll}{17}&{24}&{7}&{10}&{11}&{1}&{22}&{8}&{16}&{11}&{25}\\{17}&{1}&{14}&{12}&{11}&{16}&{25}&{1}&{6}&{14}&{11}\end{array})
19{\begin{array}{rrrrrrrrrrrrrllllllllllll}{7}&{7}&{23}&{17}&{24}&{20}&{8}&{21}&{13}&{11}&{11}&{25}&{13}&{16}&{13}&{24}&{19}\\{24}&{23}&{15}&{24}&{26}&{24}&{8}&{3}&{22}&{26}&{14}&{9}&{15}&{13}&{13}&{19}&{4}\end{array})
```




```
22{\begin{array}{rrrrrrrrrrrrrrrrrrrr}{5}&{21}&{10}&{9}&{7}&{17}&{19}&{11}&{13}&{6}&{23}&{7}&{23}&{24}\\{24}&{5}&{23}&{7}&{10}&{13}&{17}&{25}&{18}&{2}&{16}&{7}&{24}&{4}\end{array}}
| [\begin{array}{lllllllllllllll}{19}&{24}&{16}&{22}&{10}&{17}&{1}&{15}&{16}&{4}&{3}&{17}\end{array}]
{\begin{array}{llllllllllll}{4}&{19}&{7}&{13}&{7}&{14}&{13}&{12}&{16}&{22}&{15}&{22}\\{1}\end{array})
```



```
25{[\begin{array}{llllllrrrrrrrrr}{16}&{1}&{17}&{22}&{25}&{18}&{12}&{25}&{17}&{21}&{10}\\{16}&{24}&{10}&{25}&{19}&{8}&{25}&{1}&{4}&{14}&{18}\end{array})
26{\begin{array}{rrrrrrrrrrr}{24}&{15}&{19}&{9}&{19}&{9}&{2}\\{15}&{11}&{10}&{21}&{13}&{9}&{16}\end{array}}
```

Refer now to table 12 and it will be seen that categories 13 and 19 are of the greatest frequency, and they will therefore be selected for experiment. Now, it will be noted that of the 17 members of category 13,7 of them occur in line $s$. It is extremely probable that in this line the number 13 represents one of the letters of highest frequency in normal plain-text, but which one?
Assume it to represent $\mathrm{E}_{\mathrm{p}}$. Then what will the plain-text values of the other 10 members of this category be? Those 10 other members are as follows:

One in line $o$; three in line $p$; three in line $q$; two in line $r$; and one in line $u$.
Refer now to table 11 and determine what plain-text valucs are indicated, upon the assump-
ion that the seven representatives of category 13 in line $s$ represents E. Procecding aloner the tion that the seven representatives of category 13 in line $s$ represents $\mathrm{E}_{\mathrm{p}}$. Proceccing along the sline of the table to $E$, the following values will be found in the same column in which $E$ is located

> Line $o$-plain-text value is $O_{p}$ Line $p$-plain-text value is $T_{D}$ Line $q$-plan-text value is $A_{D}$ Line $r$-plain-text value is $L_{p}$ Line $u$-plain-text value is $A_{p}$

In other words, if the number 13 in line $s$ represents $\mathrm{E}_{\mathrm{p}}$, then the other members of category 1.3 would represent the letters $0, \mathrm{~T}, \mathrm{~A}$, and L , all letters of high frequency. If numerical values be assigned to these plain-text equivalents in accordance with their frequency in normal text (based upon table 2), the total value of the hypothesis that the number 13 in line $s$ represents $\mathrm{E}_{\mathrm{p}}$ is 158.6 units, determined as follows:

| Weighted Numerical Frequency Value of Category 13 upon assumption that $13=E_{\nu}$ in line $s$. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Line in text | $\begin{gathered} \text { Plain-text } \\ \text { value } \end{gathered}$ | Frequency of occurrence | $\begin{aligned} & \text { Numerical } \\ & \text { frequencal } \\ & \text { value } \end{aligned}$ | Weighted numerica frequency value |
| o | 0 | 1 | 7.4 | 7.4 |
| $p$ | T | 3 | 9. 0 | 27.0 |
| $q$ | A | 3 | 7. 2 | 21.6 |
| s | L | 7 | 3.5 | 7.0 |
| $\stackrel{s}{s}$ | E | 7 | 12. 6 | 88. 2 |
| $u$ | A | 1 | 7. 2 | 7.2 |
|  |  |  |  | Total $=158.4$ |

But suppose the number 13 in line $s$ does not represent $\mathrm{E}_{\mathrm{p}}$. Suppose it represents $\mathrm{T}_{\mathrm{p}}$. The what is the total weighted numerical frequency value of this hypothesis? It is as follows:

| Weighted Numerical Frequency Value of Category 13 upon assumption that $13=T_{D}$ in line $s$. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Line in text | $\begin{gathered} \text { Plain-text } \\ \text { value } \end{gathered}$ | Frequency of occurrence | Numerical <br> frequency <br> value | Weighted numerica frequency value |
| $o$ | A | 1 | 7.2 | 7. 2 |
| $p$ | Y | 3 | 2. 1 | 6. 3 |
| ${ }_{q}$ | C | 3 | 3.4 | 10.2 |
| $r$ | I | 2 | 7.6 | 15.2 |
|  | T | 7 | 9. 0 | 63.0 |
| $u$ | N | 1 | 7.6 | 7.6 |
|  |  |  |  | Total $=109.5$ |

The value of this hypothesis is only 109.5 units. In other words, the first hypothesis, with total of 158.4 units, is half again as probable as the second hypothesis, with a total of only 100.5 units $A$ condensed table of total values based upon the fixed hypotheses with respee to the value of the number 13 in line $s$ (viz, that it equals E, T, R, I, and N) is as follows:

| $\begin{aligned} & \text { Line } \\ & \text { ine } \\ & \text { int } \end{aligned}$ | $\begin{aligned} & \text { Freq. } \\ & \text { of. } \\ & \text { occ. } \end{aligned}$ | $\begin{gathered} \text { P.-.t. } \\ \text { yalue } \end{gathered}$ | $\begin{gathered} \text { Wtd. } \\ \text { calup. } \end{gathered}$ | $\underset{\substack{\text { P.-t. } \\ \text { value }}}{\text { cent. }}$ | Wtd. | $\begin{gathered} \text { P.-t. } \\ \text { value } \end{gathered}$ | $\underset{\substack{\text { Wtd. } \\ \text { value }}}{\text { an }}$ | $\underset{\substack{\text { P.-t. } \\ \text { value }}}{ }$ | Wtd. | $\underset{\substack{\text { P.-.t. } \\ \text { value }}}{\text { cele }}$ | $\underset{\text { velue. }}{\text { veld. }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \left(13=E_{v}\right. \text { in } \\ \text { line } s) \end{gathered}$ |  | $\begin{gathered} \left(13=\mathrm{T}_{\mathrm{v}}\right. \text { in } \\ \text { line } s) \end{gathered}$ |  | $\begin{aligned} & \left(13=R_{\mathrm{p}}\right. \text { in } \\ & \text { line } 8) \end{aligned}$ |  | $\begin{gathered} \left(13=I_{p}\right. \text { in } \\ \text { line } s) \end{gathered}$ |  | $\begin{aligned} & \left(13=N_{\mathrm{p}}\right. \text { in } \\ & \text { line } s) \end{aligned}$ |  |
| $o$ | 1 | 0 | 7.4 |  | 7. 2 | E | 12.6 |  | 3.3 | z | 1 |
| p | 3 | T | 27.0 |  | 6.3 | W | 4. 2 |  | 1.5 | U | 9. 0 |
| $q$ | 3 |  | 21.6 |  | 10.2 | G | 5. 4 | T | 27.0 | W | 4.2 |
| $r$ | 2 |  | 7.0 |  | 15.2 | B | 2. 2 |  | 25. 2 | 0 | 14.8 |
| $s$ | 7 | E | 88.2 | T | 63.0 | R | 58.1 |  | 54.6 | N | 53.2 |
| $u$ | 1 | A | 7.2 |  | 7. 6 |  | 2 |  | 1 | R | 8. 3 |
|  |  |  | 158.4 |  | 109.5 |  | 81.7 |  | 111.7 |  | 88.6 |

It is seen that the first hypothesis is by far the most probable one, and it will be assumed to be correct. The plain-text values derived from it are at once inserted in the text, wherever a member of category 13 is present
Now refer to table 13, and particularly to the prefixes and suffixes of the numbers of catecrory 13. They are as follows:
$13\left\{\begin{array}{rrrrrrrrrrrrrrrrr}23 & 22 & 3 & 1 & 14 & 26 & 23 & 9 & 16 & 24 & 11 & 3 & 21 & 1 & 19 & 19 & 3 \\ 15 & 3 & 12 & 21 & 19 & 2 & 21 & 1 & 22 & 11 & 19 & 24 & 1 & 12 & 19 & 6 & 15\end{array}\right.$

A member of category 19 occurs twice as a prefix, and three times as a suffix, five times in all. Find them in the cipher-text. They are as follows:
Column. $\qquad$ $\begin{array}{ll}\text { F } \\ 3 & 19\end{array}$ $\qquad$ -- K
-13 $\begin{array}{cc}\text { K } & \text { L } \\ 13 & 19\end{array}$
Column - --
Line $s$ $\begin{array}{cccc}\text { Y } & \text { Z } & \text { A } & \text { B } \\ 19 & 13 & 19 & 13\end{array}$

It is certain that in the line $s, 19$ represents a consonant, $R, N, S$, or $T$, most probably $R$, on account of its frequency of combination with E . Assume then, that in line $s$, number 19 represents $\mathrm{R}_{\mathrm{p}}$. What will the other diagraph of 13 and 19 in line $q$ be? Referring to table 11, it will be found that if 19 in line $s$ of the table is $R_{p}$, then in line $q$ it is $G_{p}$, thus giving the digraph 13-19 in line $q$ of the text the value $\mathrm{EG}_{\nu}$

$$
\begin{aligned}
& \text { Assume that } 19 \text { represents } \mathrm{N}_{\mathrm{p}} \text {. } \\
& \text { Assume then } 19 \text { represents } \mathrm{S}_{\mathrm{p}} \text {. Then in line } q, 13-13-19=\mathrm{E} \mathrm{~W}_{\mathrm{p}} \text {. } \\
& \text { Assume that } 19 \text { represents } \mathrm{T}_{\mathrm{p}} \text {. Then in line } q, 13-19=\mathrm{E} \mathrm{C}_{\mathrm{p}} \text {. }
\end{aligned}
$$

cre is not much choice to be made based upon any of these
y of these hypotheses. Perhaps more Forlt can be gained by determining what all the values of 19 would be upon the following hypotheses:

| $\begin{gathered} \text { Line in } \\ \text { text } \end{gathered}$ | Freq of occ. | $\begin{gathered} \text { p.t. } \\ \text { value } \end{gathered}$ | Wtat. value | $\begin{gathered} \text { P.-t. } \\ \text { value } \end{gathered}$ | Wtd. value | $\underset{\substack{\text { P.-.t. } \\ \text { value }}}{\text { cile }}$ | $\underset{\text { value }}{ }$ | $\underset{\substack{\text { P.-t. } \\ \text { value }}}{\text { cent }}$ | $\begin{aligned} & \text { Wedd. } \\ & \text { value } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\underset{\substack{19 \text { in } \\ s=R_{\nu}}}{ } \text { line }$ |  | $\underset{\left.s=N_{v}\right)}{(19 \text { in line }}$ |  | $\underset{s=S_{\mathrm{p}}}{(19 \text { in line }}$ |  | $\underset{\left.s=T_{D}\right)}{(19 \text { in line }}$ |  |
| $l$ | 2 | Q | 0.6 | Y | 4.2 |  | 2.8 | R | 16.6 |
| m | 4 | N | 30.4 | B | 4.4 | I | 30.4 | K | 1.6 |
| $p$ | 2 | W | 2.8 | U | 6.0 | K | . 8 | Y | 4.2 |
| $q$ | 3 | G | 5.4 | W | 4.2 | U | 9.0 | C | 9. 9 |
| $r$ | 1 | B | 1.1 | 0 | 7.4 | F | 3.0 | I | 7.6 |
| $s$ | 3 | R | 24.9 | N | 22.8 | S | 17.4 | T | 27.0 |
| $t$ | 2 |  | 2. 1 |  |  |  | 9.0 | H | 6.6 |
|  |  |  | 67.3 |  | 49.4 |  | 72.4 |  | 73.5 |

Of all these hypotheses, only two seem probable, viz., the ones which assume 19 in line $s$ to equal $D_{p}$ and $L_{p}$, respectively. They give totals which are so close together that it is impossible 0 tell which of the two is correct. But if reference is made to the cipher text, in line $t$, columns line $t$ the 19-19 combination would equal I I which is cording to the hypothesis that 19 in line $s=L$ then in line $t$ the 19-10 combination would equal $C C_{D_{p}}$, a very frequent doublet. qual $C^{C_{p}}$ a very frequent doublet Insertion of the values of 19 and 13 throughout yields the following:

## DISPATCH NO. 3

Key: BLOIS


```
BLOENO_
```







ISOEO OLALFVEFHRNZDXIXZKVBGIQPMLLines





NXOEO XINGQEDJMRWXXKRYSV,

81
Only two categories have been thus far determined. One more will be determined. Take the $C C_{D}$ in line $t$. It must be preceded by a rowel, $A, E, I$, or 0 . The category number of the letter concerned is 24 , one of high frequency. Here are the total different values for the four assumptions

| Line in text | Frequency | $\begin{aligned} & \text { P.-t. value } \\ & \left(24=A_{\nu}\right) \end{aligned}$ | $\begin{aligned} & \text { P.-t. value } \\ & \left(24=E_{\downarrow}\right) \end{aligned}$ | $\begin{aligned} & \text { P.-t. value } \\ & \left(24=I_{\nu}\right) \end{aligned}$ | $\begin{aligned} & \text { P.-t. value } \\ & \left(24=0_{D}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $l$ | 2 | G | J | M | F |
| $m$ | 3 | S | Q | E | V |
| $n$ | 1 | U | G | L | Z |
| $p$ | 1 | ${ }^{\text {c }}$ | A | I | P |
|  | 1 | Z | S | U | X |
| $s$ | 2 | Q | Y | D | K |
| ${ }^{t}$ | 1 | A | E | I | 0 |
| $u$ | 1 | F | H | S | G |
| ${ }_{v}^{v}$ | 2 | $\stackrel{R}{\mathrm{R}}$ | T | O | S |
| $w$ | 1 | H | U | P | T |

It is unnecessary to establish weighted frequency values, for it is obvious that the third assumption is by far the best in its results. Insertion of these values at once enables assumption to be made for certain words in the plain-text. For example, the dispatch is seen to start with the combination- 0 M ; which suggests the trigraph C 0 M , or S 0 M , for the becrinning of the dis On comber a little more experiment is necessary and the whole dispatch becomes readily solvable. It is as follows: experiment is necessary and the whole dispatch becomes readily solvable. It is as follows:

## ISPATCH NO.

Key: BLOIS



JNIRNLIFKVORARBVZUGVACCNBTLDE

Y L P C W T OLQDVHAZ Z G Z P G J PFERM Q Linen

UDPKFKQEMDSODLMOKRTDUVCANL Line O

Z Q B ORWIUPFMQOOGXMTMIJMVUBZ Line p

GAHPNGQRJFTLSIPNLWCKIETHIK Lineq

OSEROIBJOPHXSVXGLYUFYAELGK Liner

OLALFVEFHRNZDXIXZKVBGIQPMLLincs

JTOEORYHAQHQUGQXOUKCMPAQURNZEAC Line

KUOEO XNTXICLRSZOAAPHBIKSDCHRYRS Line $\underline{u}$

LVOEO WWDYCQSKKUBJIQWQFJHNUKZUSD Linev

RIBNWMSCSFMNHQDUPPUQLUURAH Line $\underline{W}$

N X O E O $\left.X_{15}\right|^{N}$ G Q E D JMRWXXKRYSV

Line $\underline{x}$

Section XIII

## RECONSTRUCTION OF OTHER ALPHABETS

Preliminary requirements to the reconstruction Complete keys for the dispatches herein analyze
$\begin{array}{r}\text { Par. } \\ 64 \\ \hline\end{array}$

| 64 | P |
| :--- | :--- |
| 65 |  |
| P |  | Proce

Proce Procedure in reconstruction of AL Par.
68
of consteconstructio
on.-Having reconstructed Alphabets 1 and 5 , it is obrious that Aplabets 2, 3 , and 4 could be reconstructed, then any message could be solved directly from the sliding strips, providing the full keyword for each message were known. It will now be shown process can be accomplished
65. Preliminary requirements to the reconstruction.-It may be stated at the outset, that prerequisite to the reconstruction of Aphabets 2,3 , and 4 is a innowedge of the complete key setting for a certain number of dispatches. This is the case eren though dispatches may be onved by detailed analysis with only a knowledge of the setting of LAW, CIV 1, 3, 5, and RAW. The principal purpose of reconstructing AL2, 3 , and 4 , is to climmate the necessity for this detailed analysis along the lines indicated in the preceding sectio

Another prerequisice to tor reconstruction is the possession in the set of dispatehes of certain ones enciphered by means of specific relative settings to be described below. Lacking such messages, the process camnot be accomplished, but failure to have the requisite dispatches, when a considerable amount of them are available for study, would be rather rare, as will be apparent when it is stated that in the set of but 10 dispatches herein studied there were found three cases hich met the required condition
66. Complete keys for the dispatches herein analyzed.-It was stated in paragraph 47解 X , that the key settinge for $\mathrm{CH}^{2} 2$ and 4 were not indicated for the set of 10 dispatches prepared by the Code and Signal Section Yavy Department. It may be well to state the reasons therefor:

It has been shown that the permutations of the automatically displaced CW 1,3 , and 5 yield an enciphering key of 17,576 letters. Manual displacements of CW 2 and 4 yield 676 sets of such keys. The theory behind the secreey of CWI and 4 was that each station could be ussivened a difceren mia of her beltines for these two wheels and thus aroid the accidental encipherment of two dispatches by different stations, in exactly the same key

Now from a consideration of what has gone before, it will be obvious that such a procedure would be more or less futile. Firstly, it would really make the solution of dispatches from the same station easier, because all dispatches originating from it would be in the same setting as regards CTV2 and 4. Secondly, in order to communicate with any station, a special key-setting code sliowing what the settings for each station are, would have to be at hand at every station, and this list would have to be changed and distributed frequently, entailing many practical difficinties. Thirdly, so far as the writer can see, the chances of two stations enciphering two messages by carly the same key (or seven letters, if given complete) would be somewhat remote, being in the ratio of $1: 26^{\text { }}$ or about 1 in 10 billion; even if it did occur, the solution of only 2 or

3 messages in identical keys, by the superimposition method, would be utterly impossible. At least 50 such dispatches would be a minimum for that method of solution. The chances for a set of stations selecting on the same day the same keyword 50 times are exceedingly remote. Usage of the machines in the military service would require that the entire keyword of seven letters be indicated for each message, so that every station could communicate with every other one without reference to any code book giving key settings, or to any secret list giving the settings for any cipher wheels.

With this in mind, it was desirable to see how difficult the reconstruction of AL2, 3, and 4 would be under such circumstances. Having first satisfied himself that such a reconstruction is absolutely impossible without a knowledge of the key settings applicable, and having already recovered and demonstrated the plain text of all ten test dispatches the writer requested the Code and Signal Section to indicate the key settings for CW2 and 4 for the 10 messages. He was informed that the setting for CW2 was the same as for CW3, that for CW4 the same as that for CW 5, in each respective keyword. For example, in the keyword AGRAM (Dispatch No. 1), CW2 was set to R, and CW4 to A. The list of complete keys is therefore as follows:

| Dispatch | Kieyword | Setting |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | LAW | CW1 | CW2 | CW3 | CW4 | CW5 | Raw |
| 1 | AGRAM. | A | G | R | R | A | A | M |
| ${ }_{2}$ | COBAN | C | 0 | B | B | A | A | N |
| 3 | BLOIS. | B | L | 0 | 0 | I | I | S |
| 4 | AGANA | A | G | A | A | N | N | A |
| 5 | CUNEO | C | U | N | N | E | E | 0 |
| 6 | DOVER | D | 0 | V | V | E | E | R |
| 7 | GENOA. | G | E | N | N | 0 | 0 | A |
| 8 | HAGUE | H | A | G | G | U | U | E |
|  | MONTE | M | 0 | N | N | T | T | E |
| 10 | NEPAL | N | E | P | P | A | A | L |

67. Study of keys.-This list was then carefully examined to find two messages which meet the following requirement, viz, that as regards $\mathrm{CW} 2,3$, and 4 there should be a case in which the two dispatches should have the same pair of key letters indicating the initial setting for CW2 the two dispatches should have the same pair of key letters indicating the initial setting for CW2
and 3, or for CW3 and 4. The purpose in finding such messages is, naturally, to be able to comand 3, or for CW3 and 4. The purpose in finding such messages is, naturally, to be able to com-
pare the cipher resultant of two specific cases wherein the differences in the cipher resultants for pare the cipher resultant of two specific cases wherein the diferences in the cipher resultants for
the same plain-text letter will be due solely to the displacement of a single cipher wheel. Three dispatches were found to conform to this requirement, viz, nos. 5, 7 , and 9 . In these three mes sages CW2 was initially set at N , and likewise CW3, although of course the similarity between the setting letter on CW2 and 3 was only a condition arbitrarily brought about by the systen adopted by the Code and Signal Section, and is not a necessary condition to be met. The setting of CW2 and 3 in cach dispatch can he different; it is only that for the purposes of this reconstruc tion the pair of settings for CW2 and 3 for one dispatch coincide with that for CW2 and 3 for another dispatch. Considering only nos. 5 and 7 , it will be seen that whatever difference ther be in the cipher resultants for the same plain-text letter enciphered in the same position a regards CW $1,2,3$, and 5 , will be due solely in the difference in the position of CW4. In no. 5 CW 4 is set at $E$, in no. 7 , at 0 .
68. Procedure in reconstruction of AL4.-Set AL5 and RFS in juxtaposition, and prepare a strip for AL4, writing NCAL4 on the upper half of the strip. Above the AL4 strip place a sequence of numbers representing the series of contacts of the fourth bakelite separator, which is to act merely as a basis of reference, as will be explained presently

Set the strips as shown below, where E of NCAL4 is at SET to correspond with the key letter applicable in CUNNEEO, and AL5 is arbitrarily set at A

BS4
--------
 RFS.......... T T Y Y O M S S L L A

As a starting point for reconstructing MCAL4 insert the letter A under M of NCAL 4. Consider now some plain-text letter, $\theta$, which enters the LHC's of AL4 from the ninth contact of BS4. The cipher resultant, with AL5 set at A, will be $I_{c}$

Now apply the I basic cipher-text sequence of table 9 to the cipher letters of the first line of the CUNEO dispatch and try to find a coincidence between a letter of this basic sequence and a letter of the cryptogram. Thus:

CW5_...- E F G H I J K L M N O P Q R S T U V WX Y Z A B C D Line U, CUNEO_ HKWZARRPBQBIVYSMPDMQMVUDC $\underset{\substack{\text { Basic Sequence No. 4, } \\ \text { Tabie } 9 \ldots}}{ }$ I A VEKCMQOSGZJBRUPWLYOFXHDITAV

No coincidence is found. Apply the basic sequence to the next line of text:

## CW5_... EFGHIJKLMNOPQRSTUVWXYZABCD

 Line V, CUNEO_ EMZXDPIDLIAWWUBQMEZPIXISNH Basie Sequence No.4,Table 9 T I AVEKCMQOSGZJBRUPWLYOFXHDITAV

Here it will be seen that the letter $\mathrm{E}_{\mathrm{c}}$ of the cryptogram coincides with E of the basic sequence. Referring to the plain-text of the dispatch it is seen that $E_{c}$ here equals $G_{p}$. The following equation may therefore be written:

```
CUNEO. Ec, key VNNEE, =G G
```

It is necessary now to translate this $E_{c}$ to some value present in the GENOA message. Since the key setting for CW1 in the CUNEO dispatch when the particular $\mathrm{E}_{\mathrm{c}}$ under discussion was enciphered was V , and since the key setting for CW1 as regards the first line of the GENOA dispatch was F it becomes necessary to find what plain-text letter E would represent in the CUNEO message if CW1 were at $V$. Hence, reference can be made to the table of basic plaintext sequences (table 11). It will be found that:

$$
\begin{array}{ll}
\text { (CUNEO) } & \text { If } \mathrm{E}_{\mathrm{c}} \text { in locus } \mathrm{Ev}=\mathrm{G}_{\mathrm{p}} \text {, then } \\
& \mathrm{E}_{\mathrm{c}} \text { in locus } \mathrm{Ee}=\mathrm{T}_{\mathrm{p}} \text {, Tha } \\
\text { (CUNEO) } & \mathrm{E}_{\mathrm{c}} \text {, key ENNEE }=\mathrm{T}_{\mathrm{p} .}
\end{array}
$$

Refer now to the E line of the GENOA message to see if there happens to be a $T_{D}$ in the line It will be found that $T_{D}$ in locus $\mathrm{Ye}=S_{c}$. The following pair of equations is then at hand:

$$
\begin{array}{ll}
\text { CONEO. } & \mathrm{T}_{\mathrm{p}}, \text { Key: ENNEE, }=\mathrm{E}_{\mathrm{c}} \\
\text { GENOA. } & \mathrm{T}_{0} \text { hey: ENNOY, }=
\end{array}
$$

These two equations are comparable except in two terms, viz, those concerning CW4 and CW5. It is easy enough to make them correspond as regards CW5. In the CUNEO dispatch the encipherment equation applies to the case where CWF is set at E; in the GENOA dispatch, it is set at Y. The matter of reducing the second equation to the case where CW5 would be t $E$ is merely one of reference to the table of basic cipher-text sequences. Thus

$$
\begin{aligned}
\text { GENOA. If } \mathrm{T}_{\mathrm{p}} \text {, key ENNOY, } & =\mathrm{S}_{\mathrm{c}} \text {, then } \\
\mathrm{T}_{\mathrm{b}}, \text { key ENNOE }, & =\mathrm{D}_{\mathrm{c}}
\end{aligned}
$$

Now there are at hand two equations in which the enciphering conditions are identical except as regards CW4. These equations are:
CUNEO. $\mathrm{T}_{\mathrm{p}}$, key ENNEE, $=\mathrm{E}_{\mathrm{c}}$
(1)
CNOA. $T_{p}$, key ENNOE, $=D_{c}$

These equations can now be used to give information with respect to MCAL4.
Refer to the sliding strips, and set them in the following positions:
1354
AL4.-

$$
\begin{aligned}
& \text { ABCDEFGHIJKLMNOPQRSTUVWXYZA)BCDEF.. } \\
& \text { ABCDEFGHIJKLMNOPQRSTUVWXYZA,BCDEF. }
\end{aligned}
$$

AL5.-
RFS...
t will be seen that the enciphering current, in order to satisfy the first of the foregoing quations, must enter that LHC of CTh which is arbitrarily designated, merely as a reference by the number 9 of BS1. Bearing this in mind slide AI. 4 to the following position (to corres pond with the key letter 0 , for CW4 in the (GENOA dispatch)
1354.

ALA_-_\{KLMNOPQRSTUVWXYZABCDEFGHIJKLMNOP.. A BCDEFGHIJKLMNOPQRSTUVWXYZABCDEF..
AL5
RFS.
HRYMS LAJIWCKUQFNVXEDPGTBZOHRYMSL..

It will be found that in order to produce the cipher letter $D_{c}$, given by the second equation, it will be necessary to insert the letter P under W of NCALA. Thus:

BS4

AL4.

AL5.
RFS.

|  |
| :---: |
|  |  |
|  |  |

This means that in NCAL4 the letter P must be situated, relative to the letter A, in the position A.

Now move AL4 back to the key-letter E position, and AL5 back to the key-letter A position. Thus:
BS4.- $\qquad$
AL4_ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEF...

AL5.-ABCDEFGHIJKLMNOPQRSTUVWXYZAB.. HRYMSLAJIWCKUQFNVXEDPGTBZOHR...
RFS.TYOEUMXDFJQVKWBNSHCILRZAGP

Assume a plain-text letter, $\theta$, which in encipherment finally enters that LIHC of CW4 designated by the number 19 in BS4, beneath which is the newly found value ${ }_{p}^{W}$ of ALA. The cipher resultant of $\theta_{\mathrm{p}}$ would be $Q_{c}$, when CW5 5 is at $A$

Now apply the $Q$ basic cipher-text seguence to the CUNEO dispatch, and try to find a coincidence between a member of this seguence and one of the cryptographic text. The following case will be found

$$
\text { CUNEO. } \mathrm{L}_{\mathrm{c}} \text {, key ZNNES, }=\mathrm{U}_{\mathrm{p}}
$$

13y reference to the table of basic plain-text sequences, table II:

$$
\text { CUNEO. If } \mathrm{L}_{\mathrm{c}} \text {, key ZNNES, }=\mathrm{U}_{\mathrm{p}} \text {, then }
$$

$$
L_{c}, \text { key ENNES, }=H_{p}
$$

Referring to the (aENOA message, line E, there is no case where the letter $\mathrm{H}_{0}$ occurs. But this does not put an and to the investigation by any means. It is merely necessary to translate the equation into terms of an $F$ line of the GENO $i$ dispatch, and this can be done by means of the table of basic plain-text sequences (table 11). Thus:

CUNEO If $\mathrm{L}_{\mathrm{w}}$ key FNNES $=\mathrm{H}$ then

$$
\mathrm{L}_{\mathrm{c}}, \text { key } \mathrm{FNNES},=\mathrm{E}_{\mathrm{p}} .
$$

Referring to line $F$ of the GENOA message, it will be found that

$$
\text { GENOA. E } \mathrm{E}_{\nu} \text {, key FNNOI, }=\mathrm{X}_{\mathrm{c}}
$$

It follows, therefore that
GENOA. $\mathrm{E}_{\mathrm{p}}$, key FNNOS, $=\mathrm{U}_{\mathrm{c}}$
(4)

Putting together the two basic equations
CUNEO. $\mathrm{E}_{\mathrm{p}}$, key FNNES, $=\mathrm{L}_{\mathrm{c}}$
CUNEO. $\mathrm{E}_{\mathrm{p}}$, key FNNES, $=\mathrm{L}_{\mathrm{c}}$
GENOA. $\mathrm{E}_{\mathrm{p}}$, key FNNOS, $=\mathrm{U}_{\mathrm{c}}$
(3)

Set the sliding strips to the following position:
BS4.
ABCDEFGHIJKLMNOPQRSTUVWWYZABCDEF


It will be seen that equation (3) is satisfied, in that $\theta_{\mathrm{p}}$ (which here enters into AL4 from th neteenth contact of BS4) equals $\mathrm{L}_{c}$.

Then moving AL4 to the following position
BS4
$\qquad$
LL4_

and tracing $U_{c}$ backward, it will be seen that $F$ must be inserted under $G$ of NCAL4 to satisfy equation (4). This gives the following placements in MCAL4:

As practice, let the reader corroborate the following cases

$$
\text { CUNEO }\left\{\begin{aligned}
C_{c}, \text { key WNNEN, }= & 0_{p} \\
C_{c}, \text { key ENNEN, }= & R_{p} \\
& R_{\mathrm{p}}, \text { key ENNOR, }=P_{\mathrm{c}} \\
& R_{\mathrm{p}}, \text { key ENNON, }=J_{c}
\end{aligned}\right\} \text { GENOA }
$$

Result: In AL4: Place $K$ under $Q$ of NCAL4.
CUNEO $\left\{\begin{array}{l}M_{c}, \text { key VNNEF }= \\ M_{c}, \text { key FNNEF, }=P_{p} \\ P_{p}\end{array}\right.$
$\left.\begin{array}{l}\mathrm{P}_{\mathrm{p}}, \text { key FNNOR },=\mathrm{T}_{\mathrm{c}} \\ \mathrm{P}_{\mathrm{p}}, \text { key FNNOF, }=\mathrm{F}_{\mathrm{c}}\end{array}\right\}$ GENOA

Continuation of this process yields the following

Only half of the MCAL4 has been reconstructed, and it would seem as if nothing further can be done because one of those irritating circumstances in alphabet reconstruction, where only half of the cycle can be recovered, has here been encountered: the key settings for CW4 for the two messages are an even number of intervals apart, and only two half-cycles can be recovered.

It would, of course, be possible to construct the two halves independently, and then try to assemble them correctly, but the process of assembling is apt to be very difficult. Another way can be and was chosen out of the difficulty.

If only one letter of the second half-cycle can be inserted in its proper position, then recon truction of this second half-cycle can be completed by reference to the same two messages CUNEO and GENOA. But how can the correct position of this single letter be determined

It will be remembered that there was another dispatch in which CW2 and 3 were at the sam key settings as in the CUNEO and GENOA dispatches, viz, the MONTE, Dispatch No. Fortunately, the two key settings for CW4 in the CUNEO and MONTE messages are an od number of intervals apart ( E to $\mathrm{T}=15$ intervals), so that any value derived from the applat of a placement in AL4 from the CUNEO message will be sure to initiate the second half of the cycle at the proper point.

Set the strips to the following positions:
BS4.-.-


Assuming a current to enter AL4 from the ninth contact of BS4, the cipher letter would be $\mathrm{T}_{\mathrm{c}}$ Applying the T basic cipher-text sequence to the CUNEO dispatch, the following case is found

EFGHIJKLMNOPQRSTUVWXYZABCD
EMZXDPIDLIAWWUBQMEZPIXISNH
Line 2, key V $\qquad$ XIGMBJAUVK.

Relering to pain text of the dispatch, it is seen that $M_{c}$ in the position shown represents $C_{D}$. This yields the following

CUNEO $\left\{\begin{array}{l}M_{c}, \text { key VNNEF },=C_{D} \\ M_{c}, \text { key ONNEF },=N_{D}\end{array}\right.$
$\left.\begin{array}{l}\mathrm{N}_{\mathrm{D}}, \text { key ONNTA, }=Y_{c} \\ \mathrm{~N}_{\mathrm{p}}, \text { key ONNTF, }=I_{c} \ldots(2)\end{array}\right\}$ MONTE

[^0]Place the sliding strips to correspond with the first equation; thus:


On tracing the path taken by the current, backward from RFS, it is seen that the thirteent ontact in $\mathrm{BS4}$ is the one involved.

Now moving the the key letter $T$ (for MONTE), and tracing $I_{c}$ backward, it will be ound that Q must be inserted under F of NCAL4. Thus:

From that point on, the two original CUNEO and GENOA dispatches may be used to - From that poirt on, the two or

$$
\text { CUNEO }\left\{\begin{aligned}
A_{c} \text { ley UNNEJ, }= & H_{p} \\
A_{c}, \text { key FNNEJ, }= & M_{p} \\
& M_{v} \text {, key FNNOO, }=T_{c} \\
& M_{p}, \text { key FNNOJ, }=H_{c}
\end{aligned}\right\} \text { GENOA }
$$

Result: In ALA: Place L under $\Gamma$ of NCALA.
Continuation of this process soon yields the complete alphabet, which is as follows:
UA. $-\ldots\left\{\begin{array}{llllllllllllllllllllllll}A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T & U & V & W & X \\ Y & T & I & Q & F & N\end{array}\right.$
69. Procedure in reconstruction of AL3.-Having reconstructed AL4 the next thing to do is to reconstruct $A L 3$. For this, after $A L A$ (or $A L 2$ ) has been reconstructed, only one dispatch is necessary, providing CW3 has been automatically displaced during the course of its encipherment. The GENOA dispatch was employed.

```
The following equations may be studied
    Since \(X_{c}\), key FNNOD, \(=S_{p}\), then
        \(\mathrm{X}_{\mathrm{c}}\), key LNAOD, \(=\mathrm{T}_{\mathrm{T}}\)
```



```
        \(\mathrm{T}_{\mathrm{p}}\), liey \(\mathrm{NOOI},=\mathrm{H}_{\mathrm{c}}\), henc
```

        \(T_{p}\), key LNOOD, \(=T_{c}\)
        (2)
    Here there are two encipherments of the same plain-text letter, $T_{\mathrm{p}}$. In the first case CW'3 was at N , in the second at O , with all the other cipher wheels at the same positions in both cuse Again a set of sliding strips is arranged, with AL4, AL5, and RF'S as the knowns, AL3 the unknown which is to be reconstructed. As before, a series of numbers representing the contacts of a bakelite separator, BS3, may be used merely as a basis of reference

Referring now to the foregoing equations, since in (1) AL3 is at N, and AL4 at 0 , with AL5 at D (key LNNOD), the cipher letter $\mathrm{X}_{\mathrm{c}}$, when traced backwards from RFS brings one to R in NCAL3. Assume as an arbitrary starting point that the current for this letter $\mathrm{T}_{\mathrm{p}}$ enters CW3 from the 20th contact of BS3. The letter G of NCAL3 is found beneath this contact, and hence $R$ may be inserted under the G. Thus


Now for the second equation: $T_{p}$, key LNOOD, $=T_{c}$. The current will again enter ALS from contact 20 of BS3, but ALA will now be at 0 . Tracing the cipher letter $T_{c}$ backward from RFS, it will be found that B must be inserted under H of NCAL3 to satisfy the equation. Thus:

BS3.

114 $\quad 0 \dot{P}$ QRSTUVWXYABCDEFGHIJKLMN



Then AL3 is again slid back to key letter N, AL5 back to A, and the new placement ${ }_{B}{ }_{B}$ is used as a basis for determining the next insertion in MCAL3. Thus:


The new value ${ }_{B}{ }_{B}$ when traced through yields $K_{c}$. Referring to the cryptogram and applying the K basic cipher-text sequence (table 9) to the text, the first coincidence is found in locus Uh, where $D_{c}=F_{p}$.

The following equations result
GENOA Since $D_{c}$, key HNNOU, $=F_{p}$, then
$\begin{array}{cc}\text { GENOA } \\ \text { (1st part) } & D_{c}, \\ & \text { Bey Lince } \\ D_{c}, & \text { key } \\ \text { But }\end{array}$
GENOA
(2d part)
$\mathrm{C}_{\mathrm{p}}$, key LNOOU, $=\mathrm{V}_{\mathrm{c}}$. . (2)
These two equations are now used to give an additional placement in MCAL3

Referring to the strips, the following is the new result:

BS3
AL3.

This process is continued until the entire sequence in MCAL3 is established, yielding the following

70. Procedure in reconstruction of AL2.-Having at hand AL1, 3, 4, and 5, it is now a very simple matter to establish AL2. Any deciphered message will do, and the process is thought to be sufficiently obvious to warrant its being passed over with a brief mention. By tracing through any plain-text letter, from LIFS, through AL1, and the resultant cipher letter backward from RFS, through AL5, 4, and 3, the continuation point of the circuit, cestablished in AL2 by the circuit from AL1 forward and from AL3 backward, becomes fixed. 'The entire AL2 is as follows:

AL2....

71. Results of complete reconstruction.--It is obvious that when all five cipher alphabets have been reconstructed, the translation of any cryptogram enciphered by means of the same horizontal permutation of the cipher wheels as that cmployed for the 10 test messages is no longer a process involving analysis, but merely one involving decipherment by manual operation
$\qquad$
of sliding strips. But suppose a different horizontal permutation is brought into play? What then?

The analysis of such messages would not be difficult. First, tables of basic cipher-text and sic plain-text sequences would be established for each one of the cipher alphabets. There would thus be five of each kind. Given a cryptogram enciphered by means of a new horize text permutation each of the five tables of basic cipher-text sequences wol substitution That table which yielded the best distribution upon the basis of sive fipher for each horizontal line of text in the dispatch would immediately show with respect to the wheels was acting as CW5. Having determined that, a similar process CW 1.
columns of the dispatch would soon show which cipher whee was acting as
Having identified the cipher wheels acting as CWI and 5 , respectively, by appy ess elucidated in section $X 1$, Solution without prelminat an the other three cipher wheels, message would soon yield. This would then show the positions could be read by reference to and thus any other message

## he strips.

These strips would be modified in minor particulars if some or all of the cipher wheels were erted in an "upside down" position. Each of the 10 tables of basic sequences would have to be tried to dermine which cipher wheel was acting as CW5; the same applies to the case of CW 1 .

## Section XIV

## REVERSE ENCIPHERMENT

Introductory statement - .-..........................
Study of electrical circuits in encipherment and decipherment. Comparison of results of the two methods using the machine ations
$\begin{array}{r}\text { Par. } \\ 72 \\ \hline\end{array}$
${ }^{\text {Par. }} 72 \mid$ So known
Solution when LFS and RFS are known
Solution when no sequences are known
4 Illustration of reconstruction of a basic --.......
75 sequence.....................................

72. Introductory statement.-When a dispateh has been enciphered with the machine set to the normal or DIRECT position, the decipherment is efficeted merely by setting the machine to the REVERSE position and depressing the leyboard leys corresponding to the cipher letters. The appropriate lumps of the lightbourd corresponding to the plain-teyt letters are illuminated. It may be well to demonstrate how this is accomplished electrically
73. Study of e'ectrical circuits in encipherment and decipherment. - The accompanying sketch (fig. 3) is intended to show how the enciphering-deciphering reciprocity is effected. The diagram applies to only one pair of letters, viz, $A_{p}=Y_{c}$, when the effective key setting of the cipher wheels is AAAAA. When the machine is set to DIRECT and key A is depressed a circuit is established as follows: From positive of battery 1, aloug conductors 2 , 3 , through closed key A, conductor 4 to movable contact 5, which, when the machine is set to DIREC'T, is in juxtaposition with fixed contact 6 . Thence the current continues along conductor 7 to a contact 8 in Lirs, hirough the cipher whecs to a contact in hrs, along conductor 10 to contact 11. Another movable contact 12, touches fixed contact 11 when the machine is set to DIRECT, and the curent contimues along conductor 13, through lamp Y, conductors 14 and 15 to negative of letter of Lamp H is lig

Now when the encipher-decipher set screw is tumed to REVERSE what happens is that the whole set of 52 movable contacts such as those shown at $5,16,12$, and 17 are shifted to the left so as to make contact with another set of fixed contacts, $26,23,31$, and 20 . Now suppose key Y is depressed. The following circuit is established. From positive of battery 1 , through conductors 2 and 18 , closed key Y , conductor 19, movable contact 17 , which is now against the left fixed contact 20, conductor 21, fixed contact 11 (the connection between fixed contact 11 and movable contact 12 being broken), conductor 10, contact 9, through the cipher wheeis (these beetio wor mon

 that when set to DIRECT the direction talen by the chent right through the cipher wheels; when set to REVERSE, it is from right to left. (Of course if the connections at the bery, when the directions taken the current . (Oh the ciper wheels will be opposite to those shown, but the directions for the DIRECT and REVERSE settings will still be opposite to each other.) This change in direction is brought about by a
reversal in the points of entry and exit provided by the conductors for the current, effected by the shifting of the contacts $5,16,12$, and 17 . The change in the direction of progress of the current through the cipher wheels is of no significance so far as the results of enciphermentants. decipherment are concerned, for after all, the direction has no effect upon the cipher through It is really only the circuit established that couns, and wherm that circuit in one direction, or the opposite, the result as expressed in the or a plain-text letter, is the same regardless of the directione
74. Comparison of results of the two methods of different solely because

of the displacements of CW5. The path taken by the current from the closed key to a contact in LFS, and thence through CW $1,2,3$, and 4, is exactly the same for all 26 depressions. The path only changes at the very end of the journey. Now suppose the macline is set to REVERSE and a message is enciphered. It will be seen, on reference to the sketch of the circuits, that depression of key A will establish a wholly different circuit from that previously noted when the machine is set to DIREC'T. With movable contact 5 against fixed contact 26, the current flows from positive of battery 1 along conductors 2 and 3 , closed key A, conductor 4, contacts 5 and 26 , conductor 27 to contact 28 . Movable contact 16 being against fixed contact 23,

29 to contact 30 in RFS. Thus a wholly different path is provided for the current into the cipher wheels than was the case with the machine set to DIRECT, and the cipher resultant will be difcrent from that produced at the DRECN setting. The current may emerge at contact 32 in LFS , thence bring the current to contact 30 in RFS, the cipher resultants will all be different due to two causes: Firstly to the successive displacements of CW5, and secondly, to the effects of this displacement upon the subsequent path taken by the current through the rest of the cipher wheels. In contrast to the case in direct operation here the path changes at the beginning of the journey of the electric current through the cipher wheels, whereas in the direct method it changes at the end of the passage through the first four cipher wheels.
Now with CW1, 2, 3, and 4 undergoing no displacement, it is apparent that for every position of CW5 there is but one path through CW5, 4, 3, 2, and 1 for a current initiated by a given key. That is, there can be but 26 paths for each plain-text letter through the five wheels, or 676 paths for the entire keyboard (CW2, 3, and 4 undergoing no displacement). Then when CW1 becomes displaced another set of 676 pathis is set up. But, when messages are arranged in lines of 26 letters (as described before), the different equivalents of the same plain-text letter falling in the same column are due solely to the displacements of CW1 so long as CW2, 3, and 4 undergo no displacement. It will be remembered that in the direct method of operation the different equivalents of the same plain-text letter in the same vertical column are also due to the displacements of CTI, but the details of the cause of the difference are not the same in both cases.
75. Results of foregoing observations.- It should be clear that if the machine has been used to encipher a set of dispatches with the normal DIREC'T setting, and these dispatches have been solved by cryptanalysis along the lines indicated in the preceding sections, the solution of another set of dispatches enciphered with the REVERSE setting should offer very little difficulties. It has been shown how the table of basic cipher-text sequences and the table of basic plain-text sequences can be reconstructed from an analysis of the former text (DIRECT operathon. He he method is strictly andorens to that described in the preceding section, with modifications necessitated by the differme in the alicability of the table of besipher and phan sequences. For in the direct method the cipher letters finally emerge from the RFS, and have been spoken of as helonging to the talle of basic cipher-teyt sequences; in the reverse method, the cipher letters emerge from the LFS, and must therefore be considered as belonging to the table of basic plain-text sequences applicable to the direct method. In order, however, to avoid a complete reversal of terminology, no change will be made in the names designating the two tables, and it will be understood that in the reverse method the plain-text letters come from the table of basic cipher-text sequences of the direct method, and that the cipher letters come from the table of basic plain-text sequences of the direct method. As before, the letters composing the text are distributed into 26 classes. Identification of 2 or 3 classes is made by recourse to principles of frequency and repetition. Then identification of the remaining classes is made by filling in the skeletons of words suggested.
76. Solution of illustrative example.-An example will serve to make the process clear. The illustrative alphabets employed in the first few pages of this paper (p.10) will be used, in
connection with which the table of basic cipher-text sequences applicable was shown as table 1 page 19. The table of basic plain-text sequences for CW1 is as follows:
table 14.-Table of basic plain-Text sequences for Cw of illustrative
(LPIABETS (P. 10)

$$
\begin{aligned}
& \text { DGZTKXFASBYRPLWUQNEMHJOCVI } \\
& \text { DGEUSNWAPXVIZLFDMGRKQYOHBCT } \\
& \text { UHOPQVZKRCJBAMXGFEWTNIYSLD } \\
& \text { TNJMCAKSFLRUWGQOBVEPIHZXDY } \\
& \text { BVPJDCLYERTHXOMKSWZQFNUIAG } \\
& \text { LTSGMDNHCEOWZXPJAUFRBCMYGFM } \\
& \text { CWRKZMTUPLDNVQEAYBOIV S } \\
& \text { EJMHPBCQDSZTKOFINXGNMVMKOU } \\
& \text { YATFWLXIZJGCRBSCVTZLYQFDRKP } \\
& \text { AUXOJMEBGHSINCRTXSJOZPLNYW } \\
& \text { FMDHEIUGAKQCHCRLDKIWSGVJRQ } \\
& \text { SOLERNYFGMUJQVTXWIDAKZPHBC } \\
& \text { SOLERNRGZWLQEHAFCOBKPMJNTX } \\
& \text { RIQXBHS JYWNKOEVZDLAFGCTPUM } \\
& \text { WRVCAGHMTZBSDPOEUYXJKQILNF } \\
& \text { HQKNXYIVBACGFRLDOMPSEUWZTJ } \\
& \text { MKILYEWONPADGJHZRTCUVXBFQS } \\
& \text { MK I L } \\
& \text { GLWBXPORVQFYUIJHKDMCAENTSZ } \\
& \text { XCEUOQRWLDVFIYKBZPNHTAGMJS } \\
& \text { PFAYGJBTINXEMKUWHQSDRLCO } \\
& \text { ZPHWI OMEQGFLSDYRVJUBCMKACN } \\
& \text { QXBVUZDLOTHPINWSJCREMKFYGA }
\end{aligned}
$$

 Now allocate the cetters of the sequences. For example, beginning with the first $m$ ef table 14 , falls in column 6 . The next letter above, it will be found that the letter $N$, in line $m$ of tahe 14 , and in on. The finst letter of the of the dispatch, P , is found in column 23 , hine $m$ of tass 14 , because $H$ occurs in column 14, line $n$ second line of the dispate,, , of the table. The process is the first two lines of the dispatch are shown herewith as an example of the process.
m $\quad \begin{array}{llllll}N & P & R & E & T & M \\ 6 & 23 & 5 & 4 & 15 & 10\end{array}$


- F Q U S K M D G Y W I K W Q C N P N J R H Q N C H W
p GIXOZVJBLRVDJGZCBCKEMAPUVY
q A L C OUIPRWOZEEMNNUDGAWDJVKG
r S BUYAXCRHBLGKYEWCHLPEWUNJZ
s EKDJBNTMPODJZTFHSLQFDLWRHF
t P Y Y K O K K D F Q Y J A Z G B O YREYXXXRF
u Y G NRXIHRPIBIWZFFKULUKXXAQA
v D U XHLBSKCHORRDNONYRJQSPONX
W WPACQVWNAKFVNYFFUYNOOADVDR
x NPQJDRHAZPRRABFJRHLOTFTVTL
y E J ZWNQGFNIP
Now it is obvious that there exists hetween members of the same category the same relationship here as was found to be the case in the study made of the dispatch solved in section XII. In the latter case it was observed that the determination of the value of a member of any class led directly to the determination of the value of any other member of the same class, through the intermediacy of the table of basic plain-text sequences. In this case the letters of the cryptogram having been distributed into classes according to their location in the table of basic plain-text sequences it follows that the relationship between members of the same class can be found by recrence to the table of basic cipher-text sequences. For example, cet us assume that the sio in line 14 of teple 1 The class in whe class 6 in line $n$ of the dispatch, viz in locus $D_{n}$ and locus $O_{n}$. Refer now to line 14 of table 1 and find the letters in columns $1 /$ and $(1$. They are $N$ and $F$, respectively. This means that if and find the letters in columns $N_{s}=M_{n}$ in locus $\operatorname{Vm}$, then $R_{c}$ in locus $D n$. Thequals $N_{n}$ and $R_{c}$ in locus $O n$ equals $F_{p}$. Similarly if $N_{e}=M_{\mathrm{p}}$ in locus m , then $\mathrm{R}_{\mathrm{c}}$ in locus $D n$ equals $\mathrm{N}_{\mathrm{p}}$ and $\mathrm{R}_{\mathrm{c}}$ in locus On equals $\mathrm{F}_{\mathrm{p}}$. Similarly to table 1. It is only necessary that the correct value of one member be assumed. It was shown in section XII how the correct, or most probably correct value can be selected by weighted frequency determinations. The same method is, of course, here also applicable. When the values of 3 or 4 categories are determined by this process as indicated above the skeletons of words soon manifest themselves. It is thought that further demonstration of the process of solution is unnecessary
. Solution when tables of basic sequences are not known.-In the foregoing case it was assumed that messages enciphered upon the reverse method of encipherment were intercepted after messages enciphered upon the direct method had been solved, so that both tables of basic
sequences were already known to the cryptanalyst. The question, of course, arises: What if he reverse method were the first that had been employed? Can the dispatches be solved The answers to these questions also assume a two-fold form, based upon two cases. First, when the LFS and RFS are known, and second, when these sequences are unknown.

8. Solution when wrs and res are known.-In the direct method of encipherment it was shown that each horizontal hine of 26 letters could be reduced to elements constituting a unique single-mised-alphabet substitution cipher. In the reverse method of encipherment, it should be clear from what has gone before, that each vertical column of 26 letters can also be reduced to elements constituting a unique single-mixed-alphabet substitution cipher. The mathematical basis for the reconstruction of ALS in the former case is also applicable to the latter case only
it is that is to be reconstructed first and not AL5. In the former case by virtue of a knowledge of the RFS, all the letters of the cryptogram can be converted into their NCAL5 c equivalents. A statistical analysis of these equivalents enables one to reconstruct AL5; this then leads lents. Astatistical analysis of these equivalents enables one to reconstruct ALs; this then leads
to a reconstruction of the table of basic cipher-text sequences; the last process enables one to resolve the letters of the horizontal lines of text into single-alphabet distributions. It follows, therefore, that in the case of reverse encipherment all the letters of the cryptogram can be converted into their $\mathrm{NCAL1}_{\text {c e cquivalents }}$ (since AL 1 is the one concerned in producing different cipher resultants for similar letters in the same column) by a knowledge of the LFS. A statistical analysis of these equiralents should lead to the reconstruction of All, this to a reconstruc tion of the table of basic piain-text sequences, and then the letters of columns can be resolved into single alphabet distributions. Experiments with dispatches have shown that there are no difficulties in the method and it is thought unnecessary to go further into detail. The solution of the single alphabet columns will be discussed late
9. Solution when no sequeaces are known,- In the absence of a knowledge of the LFS and RFS, the amalysis is, of course, much more difficult, and a large volume of text is necessary, but it is by no means impractical of achievement. The mathematical theory of repetition and nonrepetition necessary to the reconstruction of basic sequences as developed in section V applies here to the cohmins of dispatches instead of the horizontal lines. If only two basic plain-text sequences can beconstar 80. Thustan
10. applicability and truth of the hypotheses outlined in the preceding paragraph, a series of 50 in which the key setting for CW1 was such that the In this series there were found 23 messages enciphered with the key settings $0-\mathrm{P}$ for CW1. These 23 pairs of lines were subjected to a careful frequency nolve For cenmple every time L occurred in the 0 line, the letter directly beneath it in the P line was tabulated. The result was as follows:

The letter L was found to occur 26 times in line 0 , and the letter Y was found to occur most often beneath L in the P line. Refer now to the table of hasic plain-text sequences (table 14), and specifically to the 0 line. Find L , and it will be seen that Y stands directly beneath it,

By taking the sequence LY as correct, and studying P-Q horizontal lines, to find what letter most often occurs in the $Q$ line beneath $Y$ in the $P$ line, it would be found to be $M$, providing a for demonstration because it conformed to the results expected on the hypothesis, but there is
no doubt whatever that given a larger volume of text, say 100 messages, the tabulations would all conform strictly to the requirements of the theory.
of the would sequences have been reconstructed by the mathematical analysis, juxtaposition be reconstructed enable one to reconstruct the LFS, and MCAL1, and then the entire table could be reconstructed. It is thought unnecessary to make this demonstration in view of its similarit sequences.
81.5
he solution of coiumnar single-mixed alphabets.-In the direct method of encipherment having and eve a form parts of words reading horizontally. In the reverse method, how columns and the repetitions can only be indicated in analyst work But the case pon in or two lines attempts to $o l v e n t a n d$ confronts the cryptanaly when It becomes uecessary that before solution en be attaine arimposed tex In this case, the largest number of lines the easier the solution. is 26 , because CW3 becomes displaced the complete key settings for a lines from different dispatches, so as these or all the superimposed lines. The ciper equings and 4 are the same e reduced to a common basis through the reconstructed and sequences, and 26 different mixed alphabets. If all messares emanating f.
pres and superimposition by the manufacturers, the problem of finding a sufficient amount of text for different settingsecomes simpler, for then the messages from that station can show only 26 upon this scheme there turned out to be For example, in in the same of 2 , and 4 . These 10 messages vielded approximately 65 lines of superimped tes

a been cedect the common terms, by means of thasic plain-teyt sequences.
82. Steps thereafter.-Lfter solution by superimposition has been achicued the struction of the table of cipher-text sequences would be a simple matter, and would follow the
 then be in asion then be in a position to solve all dispatches directly by means of the sliding alphabets.

## Section XV

## MISCELLANEOUS

Cause of repetitions in the basic sequence $\begin{array}{r}\text { Par. } \\ -83 \\ -8 \\ \hline\end{array}$

Procedure followed in test 85
86 Recovery of cipher alphabets from a small amount --------------------- 84
of decipherad text.---
 33. Cause of repetitions in the basic sequences.-It has an waydable phenomeno repetition of at least one letter in each basic cipher-ter whe the
in this machine. It will now be shown whe the It may be stated that whenever the interval between two that is, NAL5, then a repetition the interval between these for example, AL5 of the illustrative of one letter in each b

## 

Consider the pair of letters B.....H in MALs. The interval between them coincide with the interval separating them in NAL5. Therefore, whatever cipher letter is produced for $\theta_{0}$, whein $B$ of NAL5 is the entering point for the enciphering current, $\theta_{\mathrm{p}}$ if again enciphered at i.x letters removed from its first occurrence must involve $H$ of NAL.5, and thus the same ciph letter will be produced for the second $\theta_{\mathrm{p}}$ as foir the first. Note the following diagrams
NAL5_ HIJKLMNOPQRSTUVWXYZABCDEFGHIJKL MAL5. PLJUXZGKOBTWCVMHEQNFRISYADPLJUX RFS.

$$
\text { TYOEUMXX } \mathrm{C} \text { F J QVKWBNSHCILRZAGP }
$$

NAL5_ NOPQRSTUVWXYZABCDEFGHIJKLMNOPQR MALJ GKOBTWCVMHEQNFKLSYADPLJUXZGKOBT RFS.- TYOEUMXDFJQVKWBNSHCILRZAGP

Refer now to the first basic sequeace of table 1 , page 19 , which reads as follows: Y ONDSWMAUZXFLQKGXVHRBTECJP
Note the repetition of $X$ in this sequence, at an interval six. In the same table it will be found that there is a repetition in all the seguences and that the interval between the repeated letters is always six, though the letter that is repeated is different in each sequence. The constancy in the interval is due to the fact that it is al.

Now if there were two cases in which the same phenomenon with respect to the interval between a pair of letters in MALS occurred, there would be two repetitions in each of the basic sequences; if there were three, there would be three cases, and so on. If MAL5 coincided with the normal alphabet, each basic sequence would consist of but one letter repeated 26 times Thus, the use of such an alphabet in CW5 would result in producing cryptograms completely monoalphabetic in constitution

It is obvious, therefore, that a certain amount of care must be exercised in establishing the mixed alphabet in CW 5 . Normal alphabet intervals between its letters must he aroided so far as possible

The question raises itself: Can a mixed alphabet be constructed such that the interval between ne two of its letters will coincide with their interval in the normal alphabet? The answer must be in the negative in every case in which the alphabet is one composed of an even number of elements, such as ours is. Why this is the case cannot be demonstrated here, for it would require a discission involving the Cheory of Numbers, a stitject beyont the scope of this paper. Suffice it to say that the best that can possibly be done in this case is to reduce the number of repetitions in each basic sequence to but one, the minimum possible number. This is of interest only in a purely theoretical way, for the eccurrence of several cases of repetition in each basic sequence would not materially weaken the system. ${ }^{1}$ It is evident that much care was taken in establisling the mixed alplabets of the test messages, for if examination be made itwe metwen two letters coincide with their interval in the normal alphabet
\$4. Recovery of cipher alphabets from a small amount of deciphered text. - It was not long after the author had written the explanation given in paragraph 34, section VIT, page 38, relative

 Compilation Section of this offee had previous to his entry into the military semven been in communcation with the Hebern Electric Code Co. That fire wishing to demonstrate the security of their machine sent him a cryptogram and indicated the key. So sure were they of the secrecy of the dispatch that they felt it unnecessary to break up the cipher text in to recrular groups of five letters, as is usual in practice, but left the dispatch in its original word lengths, and even stated that the text was a poem in English. They challenged Brigman to solve the message.

The following is the cryptogram as submitted to Brigman:

## fey: GORDON-Z write ELEANOR

Setting of wheels: 5-4-3-2-1, with \#4 and \#s inverted.

$$
\begin{array}{lllllllll}
\text { KB BTR } & \text { EKSMO } & \text { DG } & \text { TNS GDNX } & \text { AAT } & \text { XCN } & \text { ICA } \\
\text { IDUSEA } & \text { AJEF } & \text { HI } & \text { RGZ } & \text { TKCD } & \text { FP } & \text { AQWDJ } & \text { YD } & \text { MON } \\
\text { ZK DA } & \text { JGE } & \text { ONW } & \text { HXTCHQC } & \text { WOSG } & \text { WTMCP } & \text { BN } & \text { RF } \\
\text { GUUKHEJ } & \text { II } & \text { XHR } & \text { WARHVH } & \text { FQ } & \text { QIKCN } & \text { HGBQLY }
\end{array}
$$ PWVHHROT SMHLME PHGEEPNFY

${ }^{1}$ In an alphabet consisting of any odd number of elements, mixed alphabets can be constructed so that in
 interval repectitions, Technical Paper of the Signal Intelligence Section, 1934.

He fitted the following poem to the text, word for word, hy their lengths, after a long search through various books of quotations

```
MB
IDUSEA 
ZK
lulloll
PWVHHROT SMHLME PHGEEPNFY
```

There could not be the slightest element of doubt but that the clear text shown was correct, for the chances of finding two different pieces of clear text that would exactly fit the cipher text word-lengths, group for group, are exceedingly remote. But Brigman could not "prove" the correctness of the clear text by a cryptographic analysis. Soon after his entrance upon his
duties in this section he submitted the matter to the writer, who realized the surprisingly good opportunity afforded by such a test. It may be stated that the results were entirely successful. Both Alphabets 1 and 5 were completely reconstructed, as was an equivalent Alphabet 2-3-4, and the Table of Basic Cipher-text Sequences.
85. Procedure followed in tests.-The first thing to do was to determine whether the DIRECT or the REVERSE method of encipherment had been employed. If the former, then the test was to find the cases in which the two cipher equivalents of one pair of identical letters in one line of ext coincided with the two cipher equiralents for another pair of identical letters in the same columns but in a different line. The key indicated was, of course, of no use in the analysis, because it was evident that the key actually employed was the cipher resultant of depressing the letters of the name ELEANOR upon the keyboard, with the original setting GORDON-Z. Now it has been noted in previous work that it is most convenient to arrange the text so that the initial letter of each line gives the initial points of the various basic ciphertext sequences, in other words, so that the letter 0 of RAW is the key letter for the beginning of each line. In this case it becomes almost essential to do this, and provision must be made for it. The dispatch was accordingly written out in the manner shown below.

Note that each line after the first contains 52 letters so arranged that the key letter of ALL5 as well as of the , whaterer they be, can be made to apply to any column and the rest of the enciphered with 1 L5 at A. Then a vertical line drawn between the $Z$ and $A$ colume woul
properly align the rest of the text in lines of 26 letters; if the first letter had been enciphered at $B$, the line would be drawn between the $Y$ and $Z$ columns, and so on

## 

a $\{$

## INTHEPHOTOOFHERHEROSHECANS

( NTHEPHOTO
b $\left\{\begin{array}{l}\text { NTHEPHOTOOFHERHEROSHECANSEETHINGSTHATDONOTSHOWSOCLE } \\ \text { BBTREKSMODGTNSGDNXAATXCNICAIDUSEAAJEFHIRGZTKCDFPAQW }\end{array}\right.$
c ETHINGSTHATDONOTSHOWSOCLEARTOYOUORMEFORHEROUTLOOKEV
AIDUSEAAJEFHIRGZTKCDFPAQWDJ YDMONZKDAJGEONWHXTCHQCWO
d $\left\{\begin{array}{l}\text { RTOYOUORMEFORHEROUTLOOKEVERSEEMSTOBECOLOREDBYHERDRE } \\ \text { JYDMONZKDAJGEONHXTCHOL }\end{array}\right.$
\{ RSEEMSTOBECOLOREDBYHERDREAMSINHICHFOUKEJIIXHRWAR
e $\left\{\begin{array}{l}\text { RSEEMSTBBECOLOREDBYHERDREAMSINWHICHGOLDENSUNSHINEGL } \\ \text { GWTMCBNRFGUUKHEJIIXHRWARHVHFQQTKCWHGBQLYPWVHHROTS }\end{array}\right.$
f MSINWHICHGOLDENSUNSHTNEGI
f $\left\{\begin{array}{l}\text { MSHFQ } \\ \text { VHFQQIKCWHGBQLYPWVHHROTSMHLMEPHGEEPNF }\end{array}\right.$
$\mathrm{g}\left\{\begin{array}{l}\text { AMSENDLESSLY } \\ \text { LMEPHGEEPNFY }\end{array}\right.$
Now note the following two cases
In locus $O b, \mathrm{H}_{\mathrm{D}}=\mathrm{G}_{\mathrm{c}}$ and in locus $C^{\prime} b, \mathrm{H}_{\mathrm{D}}=\mathrm{D}$
In locus $O b, \mathrm{H}_{\mathrm{p}}=\mathrm{G}_{\mathrm{c}}$ and in ocus $C^{\prime}, \mathrm{H}_{\mathrm{D}}=\mathrm{D}_{\mathrm{c}}$
In locus $O c, \mathrm{O}_{\mathrm{p}}=\mathrm{G}_{\mathrm{c}}$ and in locus $C^{\prime} c, \mathrm{O}_{\mathrm{p}}=\mathrm{D}$
Here there are two cases such as are necessary to be found, if the DIRECT method had been employed. The reasoning behind this is as follows:

When the direct method is used, identical letters in the same line of text are enciphered by members of the same basic cipher-text sequence. Therefore, if by chance, the same basic ciphertext sequence is again employed, and if there happens to be another case of two identical letters in another line of text, and in the same columns as the letters of the first pair, and the cipher equivalents of the second case coincide with those of the first case, then it follows that the direct method had been used. If, on the other hand, the reverse method had been used, then such cases would be impossible to be produced (this follows from the mechanico-electrical relations described in the previous analysis). Hence, it seems certain that having found such a case as that noted above, the direct method was the one employed.

The same basic test can now be used to determine where the vertical line mentioned above should be drawn, to show the key setting of CW 5 . Where should this line be drawn in this case
 ine are $H_{b} G_{c}$ dean $H_{D}=D_{c}$ this much: It must full somewhe between column $C$ and 0 or between $C^{\prime}$ and $N^{\prime}$
$\Lambda$ search was then made for (1) additional cases of the nature discussed above, or (2) a case in which the requirements as to the relation between plain-text letters and cipher equivalents are not complied with. Note the following

[^1]Since the cipher letters in column $U$ are the same $\left(H_{c}\right)$ and those in column $N^{\prime}$ are different ( $K_{c}$ and $L_{c}$ ), with the same pairs of plain-text letters involved ( 0 and $E$ ), it follows that the vertical line must lie somewhere between columns $U$ and $N^{\prime}$. It has already been shown that the line cannot fall anywhere between columns 0 and $\mathrm{C}^{\prime}$ (from the preceding case) and therefore it follows that the position of the line is somewhere between columns $\mathrm{C}^{\prime}$ and $\mathrm{N}^{\prime}$

There are no cases of repetition to be found between columns $C^{\prime}$ and $N^{\prime}$ and we must therefore be content with the delimitation found thus far for the vertical line.

The next step was to try to reconstruct as much of any basic sequence as possible, from the indicated repetitions of plain-text letters and their equivalent cipher letters in each line.

The writer reasoned that it was cextremely likety that the LFS and RFS in the machine used to encipher this poem were the same as those in the machines submited for examination, because, as it has ale the chance had in mind a fixed and standard wiring for the rear plate. At any rate, it was worth the chance
to make a trial.
ollowing AL 5 was reconstructed from the partial basic cipher-text sequences resulting from a study of the text:

## 

It is not thought necessary to give the details of that work. Suffice it to say that lacking a knowledge of exactly where the vertical line should go a certain amount of experimentation was necessary before a complete MAL5 secuence could be established which would satisfy all the requirements of the text, viz, that the application of the AL5 upon RFS shourd yict. In so cipher letters shown in the cryptographic text for identical letters of the plain te... . $\mathrm{G}^{\prime}(\mathrm{p} .105)$, and that the text should be arranged as shown below. Let the reader verify that this is the case, and also verify MAL5.
a $\left\{\begin{array}{l}\text { AOCDEFGHIJKLMNOPQRSSUXZ }\end{array}\right.$

$b\left\{\begin{array}{lllllllllllllllllllllllllll}0 & T & O & O & F & H & E & R & H & E & R & O & S & H & E & C & A & N & S & E & E & T & H & I & N \\ S & M & O & D & G & T & N & S \\ G & D & N & X & A & A & T & X & C & N & I & C & A & I & D & U & S & E\end{array}\right.$
$c\left\{\begin{array}{llllllllllllllllllllllllll}S & T & H & A & T & D & O & N & O & T & S & H & O & W & S & O & C & L & E & A & R & T & O & Y & O & U \\ A & A & J & F & F & H & I & R & G & Z & T & K & C & D & F & P & A & Q & W & D & J & Y & D & M & O & N\end{array}\right.$


 $g\left\{\begin{array}{lllllll}L & E & S & S & L & Y \\ E & E & P & N & F & Y\end{array}\right.$

After MAL5 was reconstructed the entire table of basic cipher-text sequences was easily reconstructed. Following this AL1 was reconstructed by reference to the text. It was found
to be as follows:

## 

## Then an equivalent $A L 2,3$, and 4 was reconstructed, as follows:

Equivalent 2-3-4..... | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ | $K$ | $L$ | $M$ | $N$ | $O$ | $P$ | $Q$ | $R$ | $S$ | $T$ | $U$ | $V$ | $W$ | $X$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Q | W | $T$ | $B$ | $U$ | $Z$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

(These are not the "converted alphabets", but the real ones; that is, in using them, a letter of the normal component is traced to the same letter in the mixed component,

Test the following set of strips on the text, with the initial points as shown in the diagram
LFS_-.- BSXRZTKDNCHMVOLYQEUPWJAIF
AL1 $\ldots-\left(\begin{array}{llllllllllllllllllllllll}A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T & U & V & W & X \\ E & H & L & N & S & V & Z & C & F & K & M & Q & U & Y & D & J & P & R & W & A & G & I & 0 & T\end{array}\right]$
AL $2-3-4 \ldots\left\{\begin{array}{llllllllllllllllllllllll}J & K & L & M & N & O & P & Q & R & S & T & U & V & W & X & Y & Z & A & B & C & D & E & F & G\end{array}\right.$

RFS...- TYOEUMXDFJQVKWBNSHCILRZAGP
It is clear that by using these strips the correctness of the clear text has been established to an absolute degree. One error in encipherment (or copving ?) was found. The letter $H_{p}$ of the word SUNSIIINE was incorrectly designated by the letter $H_{c}$; it should have been $Y_{c}$.

It is obvious that the analyst is now in a position to solve all other messages written by means of the same cipher whecls in the same horizontal permutation. This shows how farreaching the effects of finding even a short message with its decipherment would be in actual practice.
86. Analysis by superimposition.- The method of analysis of cryptograms by recourse 10 the principles of superimposition are, of course, among the most fundamental processes in cryptanalysis, and are resorted to only when all other methods fail. In this superimposition, letters which have been enciphered by the same secondary alphabets are brought together within sufficient number of letters is included in such cole on the basis of pure frectuency. When a sufficient whmber of letters is included in such columns, solution can always be achieved, no

In the case of this machine, when the key words for dispatches are
of superimposition can be applied to this cryptorraphic system. It it known, the principles of superimposition can be applied to this cryptographic system. It is possible, of course, to
use the machine in conjunction with a code book for indicating the key words, in which case enemy cryptanalysts might have no clues as to the key word for each dispatch, providing that the code system adopted really affords the kind of secrecy necessary for the purpose. But the necessity for using a code book would constitute such a very serious disadvantage that for practical reasons it would be most advisable to dispense with such use and take chances on what information the enemy could obtain from a knowledge of the key words

It is obrious from what has gone before, that every one of the secondary alphabets of this macline can be given a number, and that every letter of every message can be allocated to the secondary alphabet to which it belongs. If a sufficient amount of text is available, it can
easily happen that 50 or more dispatches can be superimposed, thus yielding columns of 50 o more letters which then constitute the elements of single alphabet substitution ciphers. Solution of such columns is possible by recourse to the simple principles of frequency. It is unnecessary to indicate how the allocation can be made, for it will be obvious to anyone who has a thorough comprehension of the mechanics of the system of cipher-wheel movement. If every station has a different setting as regards CW2 and 4, then the traffic of the most important station may easily yield a sufficient number of dispatches for superimposition, since in this case only 17,576 secondary alphabets are involved. If the same cipher wheels and the same horizontal permutation are used for a number of days, then there would be no question about the availability of a sufficient amount of text for superimposition and solution.

DISPATCH NO. 1
Key: AGRAM. (Effective key: AGRBN)
RAW.- OPQRSTUVWXYZABCDEFGHIJKLMN
 A G R B N...... $\{$
BHRCO..... NUTXHVZSLUMLZXHXHOHYBRCLMS






$H N R C O-\ldots-\left(\begin{array}{ll}E N M O Q \\ E\end{array}\right.$



INATOPARTICIPATEFOLLOWINGJ

$M \operatorname{S} R C O \ldots \ldots\left\{\begin{array}{llllllllllllllllllllllllll}A & B & L & Z & C & J & U & C & L & J & X & S & O & U & D & L & W & U & T & A & F & I & A & R & T & U \\ A & L & L & I & A & N & C & E & B & R & I & N & G & S & R & E & A & L & I & Z & A & T & I & O & N & 0\end{array}\right.$

(111)

> Key: COBAN. (Effective key: DPBBO)

RAW．－O P Q R S T UVWXYZABCDEFGHIJKLMN気念若咅运CW5．．．BCDEFGHIJKLMNOPQRSTUVWXYZA
 E Q B B $0 \ldots \ldots\left\{\begin{array}{llllllllllllllllllllllll}U & Q & I & S & A & H & S & V & I & H & S & W & D & T & I & D & Y & A & B & J & G & T & K & K \\ E & M & M & Y \\ X & C & O & M & P & R & E & S & S & I & O & N & R & E & Q & U & I & R & E & M & E & N & T\end{array}\right.$


H T B B O＿－．．－$\left\{\begin{array}{l}X M R Y W F Z H E B B Z E B X F F W H P F V Y H F V\end{array}\right.$







Q B C B O $\ldots-\left(\begin{array}{llllllllllllll}W & P & Z & A & Q & S & T & M & K & G & H & G \\ W & O & O & R & T & H & R & E & E & W & E & E & K & S\end{array}\right.$

BL 0 J T．．．．．．

C M O E O－．．．．－


E O O U D P

FPOE O＿．．．．．\｛ Z Q B O RWI UPFFHQOGXMTMIJMVUBZ






 I TH

N X O E O－ MYY R E P
should be ZX．
＊Underined portion should be ZX．
These were errors in encipherment

## DISPATCH NO． 4

## Key：AGANA．（Effective key：AGAOC

$\begin{array}{lllllllllllllllllllllllll}\text { RAW．．．} & O & P & Q & R & S & T & U & V & W & X & Y & Z & A & B & C & D & E & F & G & H & I & J & K & L \\ \text { CWS．．．} & A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T & U & V & W & X\end{array}$音空会总急
A G A O C．．．．．．－
$\begin{array}{llllllllllll}\text { F } & S & R & U & X & M & M & F & Y & E & P & A \\ N & A & V & A & L & C & 0 & U & N & C & I & L\end{array}$
B H A A O


E K A A O


HN A A O

JPAA O．．．．．．$Z$ ZAMPKQADBRBCORPUGJIHKA JKLK
K Q A A O－．．．－－MGSPEGRESIFAIXZQFIWMADUCFM
L R A A O

The key as given was in error．It should have read AGANB instead of AGANA

Key：CUNEO．（Effective key：CUNFP）

急总号急
C UN F P．．．．
$\qquad$
 DVNEO EMZXDPIDLIAWWUBQMEZPIXISNH EWNEO． OHBCMELYWHOSTA

 HZNEO MKDQEUDKMIGEOJLRZDKNNPNYXY
 J B N E O．．．．．．WK WAVUEASULCOGRQLZWUKIKTJZ
 L D N E O $-\ldots-\left(\begin{array}{llllllllllllllllllllllllll}V & C & J & A & B & X & N & D & I & W & C & C & E & M & H & G & K & Q & Q & D & C & B & I & G & R & I \\ N & D & S & A & W & A & S & I & M & I & L & A & R & B & 0 & X & W & I & T & H & 0 & N & E & C & 0 & R\end{array}\right.$ M E N E O．．－．．－\｛ A Z E H O F O R R Z F F J O N F I V S M O Q W T Z I S
 PGOE O $\ldots, \ldots\left\{\begin{array}{llllllllllllll}H & Q & B & X & R & Z & I & I & Z & M & C & S & P & Z \\ R & K & E & D & H & A & I & G & A & N & D & H & A & I \\ G\end{array}\right.$
Nore－Underlined portions were incorrectly enciphered

DISPATCH No. 6
Key: DOVER. (Effective key: DOVFS)

 D 0 V F S.-....L P P I O O U E E Z K K S J J B X X C A F F E U U K S S D W H

$\qquad$

F Q V B O-.-.--
$\qquad$
 HSVBO_.... FEKXMRALNVRSKAESDSMTGRXSYP
 J UV B O_..... $\begin{aligned} & \text { EP P } I X V E J E B H I F G S V P X X G A Z C Q C Z S\end{aligned}$
 L W V B $0 \ldots \ldots-\left(\begin{array}{llllllllllllllllllllllll}A & N & W & W & T & N & K & V & B & Z & U & Y & R & T & P & M & R & W & P & I & C & V & Q & Z \\ A & R & O & F & C & D & M & P & L & I & C & A & T & I & O & N & S & W & I & T & H & J & A & P \\ A & N\end{array}\right.$
 N Y V B 0......-

 Q A W B 0... FK O S P P S C FABZWSNTBYBXQMGGVAN Nore,—Underlined portion was incorrectly enciphered.
 G E N P B......-

 I G N C O_..... N R OTLDHSWW GMIPBZZCGPGPRVTB (

 JYVTNDBMEQM-SAZDYAWGHC






S P O C O..... SVZWWGOYQCW JSADPSOQUYHDS FDANTONCL


Note.-Underlined portions were incorrectly enciphered.

## DISPATCH NO. 8

Key: HAGUE. (Effective key: HAGVF)
RAW..

 S Y Y G S T X XVP HAGVF-....-HAVEABSOL


K D G E O $\quad \ldots, \ldots\left\{\begin{array}{llllllllllllllllllllll}W & Y & S & X & V & B & I & W & O & A & Z & G & Z & E & J & A & D & C & K & W & Z & K \\ \text { W } & \text { I } & I \\ \hline\end{array}\right.$





R J HE O...... AG I NKL B WV X JVCVVZYKAWOPTECPY

- GUWIJKBQPWOTHSGLSYDPQUQNJ


U M HE O_,
Nore.-Underlined portion represents error in encipherment.

DISPATCH NO. 9
Key: MONTE. (Effective key: MONUF)
Raw.- OPQRSTUVWXYZABCDEFGHIJKLMN
 M O N U F.......-

NPND O_..... UUJJS LSWCRNULSAEXTGWP Z (
 QRODO_ OVDUQZTHHSYGEGKO O OHXRPMH (XIOUSTOPROCEEDASSOONASROMS
 STODO..... VEYYGODZLNMLVKUOFRXHVKIHPF THEYPROPOSESEVERALMODIFIC



VWODOS SUSKBHHFNTBYXYXDTHKEHQJ STOPMEANWHILETHEPRESENTGER





RAW.. OPPQR S T U V W X Y Z Z A B C D D E F G G H I J J K L M N

0 E Q B M......-
$\left\lvert\, \begin{array}{ll}Z & N \\ A & R\end{array}\right.$
 Q G Q D $0 \ldots \ldots\left(\begin{array}{lllllllllllllllllllllllll}E & U & Z & W & C & L & G & R & B & M & Q & K & T & C & C & G & H & V & P & T & F & A & B & X & D \\ A & P & A & N & E & S & E & P & R & O & P & A & G & A & N & D & A & H & E & R & E & A & N & D & C \\ H\end{array}\right.$


$\qquad$

T JQ D O.....--
UK Q D O.......


WM Q D O....... $\begin{array}{llllllllllllllllllllllllll}P & X & D & J & K & B & X & C & N & I & F & S & C & M & G & T & G & B & T & R & E & M & T & E \\ L & L & I & N & G & T & O & M & A & K & E & S & O & M & E & R & E & D & U & C & T & I & O & N & B & U\end{array}$

X N Q D O K L H C C C U P D Q X C P T F O B L M V Y Z R I R M V

Y O Q D O......-


ZPQD O.......


AQQD O....... | $Y$ | $C$ | $X$ | $Q$ | $T$ | $B$ | $P$ | $J$ | $D$ | $M$ | $U$ | $S$ | $I$ | $R$ | $O$ | $K$ | $Y$ | $M$ | $A$ | $D$ | $O$ | $B$ | $V$ | $U$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $N$ | $P$ | $Z$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

BRQD O.......

Note-EError in underlined portion



A ABCDEFGHIJKLM

A \begin{tabular}{l|l}
B \& S <br>
C \& <br>
\cline { 2 - 2 } \& <br>
\hline

 

\hline \& 6 \& 2 \& 7 \& 2 \& 4 \& 7 \& 3 \& 3 \& 6 \& 2 \& $B$ <br>
\hline
\end{tabular}

 \(\begin{aligned} \& \mathrm{D} <br>

\& \mathrm{E}\end{aligned} \frac{1}{1} |\)| 9 | $\frac{4}{5}$ | $\frac{4}{2}$ | $\frac{1}{1}$ | $\frac{8}{5}$ | $\frac{5}{3}$ | $\frac{5}{2}$ | $\frac{7}{3}$ | $\frac{7}{6}$ | $\frac{5}{6}$ | $\frac{6}{5}$ | $\frac{0}{2}$ | $\mathbf{D}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| E |  |  |  |  |  |  |  |  |  |  |  |  | F

G

 | I | $\frac{5}{5}$ | 4 | 4 | 6 | 3 | 5 | 5 | 4 | 2 | 6 | 4 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| J | 2 | 6 | 2 | 2 | 6 | $\frac{1}{5}$ | $\frac{1}{5}$ | 2 | 4 | 2 | 4 | 7 |

J
K 5

 M \begin{tabular}{llllllllllllll}
4 \& 4 \& 7 \& 10 \& 6 \& 1 \& 6 \& 0 \& 3 \& 3 \& 4 \& 8 <br>
\hline

 $\mathrm{O}_{1}$ A B C C D E F G H I J K L M 

\hline $\mathbf{N}$ <br>
\hline \& 3 \& 3 \& 3 \& 5 \& 2 \& 2 \& 2 \& 7 \& 6 \& 1 \& 3 \& 7 \& 5 <br>
\hline

 

\hline O \& 4 \& 4 \& 8 \& 6 \& 1 \& 2 \& 8 \& 5 \& 3 \& 3 \& 5 \& 3 <br>
\hline

 

$\mathbf{P}$ \& 7 \& 2 \& 5 \& 5 \& 3 \& 4 \& 3 \& 4 \& 5 \& 4 \& 6 <br>
\hline \& 2 \& 1 <br>
\hline

 

\hline $\mathbf{Q}$ \& 2 \& 1 \& 4 \& 4 \& 8 \& 3 \& 3 \& 8 \& 3 \& 7 \& 1 \& 5 <br>
\hline

 

$\mathbf{R}$ \& 4 \& 3 \& 3 \& 3 \& 5 \& 3 \& 5 \& 3 \& 8 \& 10 \& 1 <br>
\hline

 

\hline S \& 4 \& 6 \& 6 \& 5 \& 5 \& 3 \& 6 \& 6 \& 6 \& 2 \& 4 \& 9 <br>
\hline

 

$\mathbf{T}$ \& 10 \& 8 \& 5 \& 6 \& 4 \& 6 \& 3 \& 5 \& 4 \& 4 \& 5 <br>
\hline
\end{tabular} U

 \begin{tabular}{c|c|c|c|c|c|c|c|c|c|c|c|c|c}
$\mathbf{W}$ \& 4 \& 4 \& 3 \& 5 \& 4 \& 7 \& 8 \& 0 \& 11 \& 3 \& 0 \& 4 \& 3 <br>
$\mathbf{x}$ \& 4 \& 9 \& 4 \& 3 \& $\mathbf{7}$ \& $\mathbf{5}$ \& 6 \& 5 \& 4 \& 4 \& 3 \& 4 \& 1 <br>
$\mathbf{X}$

 

\hline $\mathbf{y}$ \& 8 \& 3 \& 4 \& 8 \& 8 \& 6 \& 9 \& 6 \& 8 \& 6 \& 1 \& 7 \& 5 <br>
\hline
\end{tabular}

| $\mathbf{z}$ | 8 |
| :--- | :--- |
|  | 3 |

N O P Q R S T U V W X Y Z | 5 | 0 | 7 | 2 | 5 | 1 | 5 | 6 | 6 | 4 | 6 | 5 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 3 |  | 0 | 1 | 5 | 3 | 2 |  |  |  |  |  |



 \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|l}
\hline 3 \& 5 \& 5 \& 8 \& 3 \& 6 \& 3 \& 3 \& 6 \& $\frac{5}{5}$ \& 8 \& 6 \& $\frac{3}{9}$ <br>
\hline \& C

 

\hline 4 \& 5 \& 4 \& 3 \& 7 \& 5 \& 4 \& 6 \& 4 \& 6 \& 6 \& 5 <br>
\hline 4 \& $\frac{5}{5}$ \& $\mathbf{E}$ <br>
\hline

 

\hline 4 \& 3 \& 8 \& 6 \& 5 \& 1 \& 8 \& 7 \& 1 \& 1 \& 5 <br>
\hline

 

\hline 4 \& 6 \& 2 \& 5 \& 5 \& 4 \& 4 \& 6 \& 5 \& 3 \& 3 \& 9 \& 5 \& $G$
\end{tabular}



 | 3 | 4 | 3 | 1 | 2 | 6 | 6 | 7 | 10 | 7 | 1 | 4 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $J$ |  |  |  |  |  |  |  |  |  |  |  |



 | 5 | 4 | 3 | 3 | 4 | 2 | 3 | 7 | 5 | 2 | 2 | 5 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | N O P Q R S T U V W X Y Z

| 1 | 4 | 7 | 4 | 4 | 1 | 3 | 4 | 4 | 3 | 7 | 3 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 1 | 3 |  | 1 | 7 | 4 |  |  |  |  |  |

 \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|}
\hline 6 \& 1 \& 3 \& 2 \& 5 \& 7 \& 2 \& 4 \& 3 \& 5 \& 4 <br>
7 \& 7 <br>
\hline \& 1 \& \& 4 \& \& \& 7 \& \& <br>
\hline

 

3 \& 1 \& 5 \& 4 \& 4 \& 5 \& 7 \& 10 \& 1 \& 4 \& 4 <br>
\hline \& 4 \& 3 \& $\frac{5}{7}$ \& 4 \& $\mathbf{Q}$ \& <br>
\hline

 $\begin{array}{lllllllllllllll}7 & 4 & 3 & 7 & 5 & 8 & 10 & 2 & 7 & 5 & 10 & 6 & 5 & R\end{array}$ 

1 \& 5 \& 3 \& 4 \& 6 \& 4 \& 6 \& 8 \& 7 \& 7 \& 5 \& 4 \& 1 \& S
\end{tabular}

 \begin{tabular}{lllllllllllllll}
1 \& 5 \& 6 \& 4 \& 4 \& 4 \& 7 \& 6 \& 7 \& 5 \& 13 \& 4 \& 4 <br>
\hline

 

7 \& 4 \& 4 \& 2 \& 8 \& 9 \& 4 \& 6 \& 3 \& 2 \& 2 \& 8 \& 3 <br>
\hline \& 4 \& 4 \& \& <br>
\hline
\end{tabular}

 \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|}
\hline 1 \& 10 \& 7 \& 4 \& 7 \& 6 \& 11 \& 4 \& 5 \& 3 \& 4 \& 2 <br>
\hline

 $\mathbf{Z}$

\hline 6 \& 5 \& 4 \& 5 \& 5 \& 6 \& 8 \& 2 \& 3 \& - \& 3 \& 9 \& 6 \& $\mathbf{z}$ <br>
\hline
\end{tabular} N O P QRETUVWXYZ

A B C D E F GH TH K M A | 9 | 2 |  | 2 | 3 | 7 | 4 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

N O P Q R S T U V W X Y Z

 \begin{tabular}{l|l|llllllllllllll}
C \& 3 \& 5 \& 5 \& 5 \& 2 \& 2 \& 5 \& 3 \& 8 \& 3 \& 5 \& 5 \& 4

 D 

7 \& 8 \& 0 \& 8 \& 4 \& 3 \& 2 \& 3 \& 4 \& 6 \& 3 \& 8 \& 5 <br>
\hline \& \& 4 \& 2 \& \& 6 \& 2 \& <br>
\hline

 

E \& 3 \& 4 \& 2 \& 3 \& 6 \& 2 \& 1 \& 5 \& 3 \& 2 \& 4 \& 1 \& 5 <br>
\hline

 F G 

\hline

 $\mathbf{4}$ H 

H <br>
\hline

 A 

\hline I \& 4 \& 2 \& $\frac{7}{2}$ \& 1 \& $\frac{3}{3}$ \& $\frac{5}{10}$ \& $\frac{4}{6}$ \& $\frac{6}{4}$ \& $\frac{1}{7}$ \& $\frac{7}{3}$ \& $\frac{3}{5}$ \& 2 <br>
\hline

 

J <br>
\hline 2

$\frac{4}{2}$ 

\hline $\mathbf{K}$ \& 1 \& 4 \& 4 \& 5 \& 5 \& 5 \& 5 \& 5 \& 2 \& 2 \& 4 \& 5 \& 2 <br>
\hline

 

L \& 7 \& 6 \& 6 \& 2 \& 2 \& 7 \& 1 \& 2 \& 3 \& 6 \& 4 <br>
\hline

 

\hline M \& 4 \& 4 \& 2 \& 6 \& 4 \& 3 \& 1 \& 4 \& 6 \& 3 \& 3 \& 1 \& 3 <br>
\hline

 ${ }_{\theta_{1}}^{1}$ A B C D D E F G H I J K L M 

\hline \multirow{2}{*}{} \& 2 \& 7 \& 4 \& 4 \& 1 \& 2 \& 7 \& 3 \& 1 \& 5 \& 8 \& 1 \& 0 <br>
\hline 0 \& 7 \& 0 \& \& \& \& \& \& \& \& \& \& \& <br>
\hline

 

\hline o \& 7 \& 6 \& 1 \& 8 \& 6 \& 6 \& 4 \& 3 \& 4 \& 2 <br>
\hline

 

\hline $\mathbf{P}$ \& 3 \& 2 \& 5 \& 3 \& 7 \& 9 \& 4 \& 9 \& 8 \& 2 \& 4 <br>
\hline

 

Q \& 3 \& 1 \& 7 \& 7 \& 5 \& 6 \& 1 \& 5 \& 6 \& 1 \& 12 \& 4 \& 4 \& <br>
\hline
\end{tabular}

 \begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline 5 \& 6 \& 13 \& 3 \& $\frac{4}{2}$ \& $\frac{6}{8}$ \& $\frac{6}{4}$ \& $\frac{6}{5}$ \& $\frac{1}{4}$ \& $\frac{9}{5}$ \& $\frac{2}{3}$ \& 4 \& 2 <br>
\hline 2 \& 8 \& 8 \& 5 \& 5 \& 6 \& 4 \& \& \& 7 \& 5 \& 5 \& <br>
\hline

 

5 \& 6 \& 13 \& 3 \& 2 \& 8 \& 4 \& 5 \& 4 \& 5 \& 3 \& 7 \& 1 <br>
\hline 2 \& 8 \& 8 \& 5 \& 5 \& 6 \& 4 \& 5 \& 7 \& 5 \& 5 \& 3 \& 4 <br>
\hline \& \& \& \& <br>
\hline

 

T \& 2 \& 8 \& 8 \& 5 \& 5 \& 6 \& 4 \& 5 \& 7 \& 5 \& 5 \& 3 \& 4 <br>
\hline U \& 6 \& 10 \& 5 \& 5 \& 4 \& 4 \& 8 \& 5 \& 4 \& 5 \& 3 \& 5 \& 4

 

$\mathbf{~ U}$ \& $\frac{6}{9}$ \& $\frac{10}{4}$ \& $\frac{5}{2}$ \& 5 \& 4 \& 4 \& $\frac{4}{2}$ \& $\frac{8}{7}$ \& $\frac{5}{2}$ \& $\frac{4}{5}$ \& $\frac{5}{4}$ \& 3 <br>
\hline \& 5 \& 4 <br>
\hline
\end{tabular}

 \begin{tabular}{l|c|c|c|c|c|c|c|c|c|c|c|c|c|}
$\mathbf{W}$ \& 5 \& 2 \& 5 \& 5 \& 6 \& 5 \& 8 \& 2 \& 6 \& 3 \& 3 \& 4 \& 3 <br>
$\mathbf{X}$ \& 5 \& 14 \& 4 \& 6 \& 6 \& 2 \& 7 \& 4 \& 2 \& 2 \& 5 \& 4 \& 0

 

\hline $\mathbf{y}$ \& 8 \& 1 \& 1 \& 7 \& 7 \& 7 \& 5 \& 9 \& 2 \& 2 \& 5 \& 4 \& 0 <br>
\hline

 

\hline 3 \& 2 \& 7 \& 6 \& 7 \& 3 \& 3 \& 3 \& 3 \& 6 \& 5 \& 5 \& 4 <br>
\hline A B C D E F G H I J K L M
\end{tabular}

 \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|l}
\hline 5 \& 1 \& 4 \& $\frac{5}{3}$ \& $\frac{5}{2}$ \& $\frac{4}{3}$ \& $\frac{7}{5}$ \& $\frac{7}{3}$ \& $\frac{5}{9}$ \& 3 \& 7 \& 9 \& 6 <br>
\hline

 $\frac{5}{3} \frac{1}{15} \frac{4}{4} |$

4 \& $\frac{2}{6}$ \& $\frac{3}{6}$ \& $\frac{5}{3}$ \& $\frac{3}{1}$ \& $\frac{9}{1}$ \& $\frac{5}{2}$ \& $\frac{4}{5}$ \& $\frac{6}{4}$ \& $\frac{8}{8}$ \& C <br>
\hline

 

3 \& $\frac{15}{3}$ \& $\frac{4}{4}$ \& $\frac{4}{4}$ \& $\frac{6}{6}$ \& $\frac{6}{4}$ \& $\frac{3}{7}$ \& $\frac{1}{3}$ \& $\frac{1}{2}$ \& $\frac{2}{4}$ \& $\frac{5}{7}$ \& 4 <br>
\hline 7 \& $\frac{8}{2}$ \& D

 

7 \& $\frac{3}{4}$ \& 4 \& 4 \& $\frac{6}{3}$ \& $\frac{4}{4}$ \& $\frac{4}{4}$ \& $\frac{7}{1}$ \& $\frac{3}{5}$ \& $\frac{3}{6}$ \& $\frac{4}{5}$ \& $\frac{7}{7}$ \& 12 <br>
\hline \& 7 \& E <br>
\hline

 F 

2 \& 4 \& 9 \& 3 \& 4 \& 4 \& 1 \& 5 \& 6 \& 5 \& 7 \& 5 <br>
\hline
\end{tabular}

 \begin{tabular}{l|l|l|llllll|l|l|l|l}
\hline \& 1 \& 0 \& 6 \& 2 \& 6 \& 10 \& 12 \& 2 \& 8 \& 5 \& 0 \& 4 <br>
\hline

 

1 \& 3 \& 5 \& 2 \& 5 \& 5 \& 6 \& 6 \& 3 \& 5 \& 2 \& 2 \& 5 \& I <br>
\hline

 

5 \& 6 \& 6 \& 7 \& 4 \& 2 \& 2 \& 4 \& 5 \& 3 \& 2 \& 5 \& 6 \& $J$

 $\begin{array}{llllllllllllllll}3 & 3 & 2 & 6 & 5 & \frac{2}{3} & 6 & 4 & \frac{5}{4} & \frac{0}{6} & \frac{2}{6} & \frac{5}{5} & \frac{6}{5} & J\end{array}$ 

\hline 7 \& 4 \& 4 \& 5 \& 7 \& $\frac{3}{2}$ \& 6 \& 4 \& 4 \& 6 \& 6 \& 5 \& 5 \& $K$

 

7 \& $\frac{4}{4}$ \& 4 \& 5 \& $\frac{7}{2}$ \& $\frac{10}{3}$ \& $\frac{5}{3}$ \& $\frac{7}{2}$ \& $\frac{1}{3}$ \& 3 \& 5 \& 5 \& $\frac{4}{2}$ \& $\mathbf{L}$ <br>
\hline 7 \& 3 \& 2 \& 3 \& 4 \& 3 \& 3 \& 2 \& 3 \& 7 \& 6 \& 3 \& 3 \& M

 N O P Q R S T U V W X Y Z 

\hline $\mathbb{N}$ \& 1 \& 1 \& 2 \& 4 \& 6 \& 9 \& 1 \& 6 \& 2 \& 4 \& 6 \& 2 \& 4 <br>
\hline \& \& 0 \& \& \& 7 \& 4 \& 5 \& 5 \& 1 \& 7 \& 5 \& 3 \& 5
\end{tabular} $\begin{array}{lllllllllllllll}3 & 6 & 3 & 7 & 4 & 5 & 5 & 1 & 7 & 5 & 3 & 5 & 4 & 0\end{array}$

 \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|l}
3 \& $\frac{8}{4}$ \& 3 \& $\frac{3}{2}$ \& 2 \& 2 \& 2 \& 5 \& 2 \& 0 \& 6 \& 3 \& 5 <br>
\hline 4 \& P \& 3 \& 2 \& 3 \& 2 \& 4 \& 4 \& <br>
\hline

 

4 \& 2 \& 3 \& 2 \& 3 \& 3 \& 2 \& 4 \& 4 \& 0 \& 5 \& 6 <br>
\hline 8 \& 7 \& 4 \& 9 \& 3 \& 6 \& 12 \& 6 \& 4 \& \& <br>
\hline

 

S \& B \& 7 \& 4 \& 9 \& 3 \& 6 \& 13 \& 6 \& 4 \& 4 \& 5 \& 5 \& 4 <br>
\hline
\end{tabular}

下 \begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c}
0 \& 6 \& 4 \& 6 \& 7 \& 7 \& 3 \& 10 \& 5 \& 6 \& 5 \& 11 <br>
\hline 1 \& 5 \& T

 

\hline 1 \& 3 \& 3 \& 6 \& 7 \& 3 \& 8 \& 2 \& 6 \& 6 \& 6 \& 1 <br>
\hline- \& 5 \& $\mathbf{U}$

 

\hline 5 \& 3 \& 3 \& 1 \& 5 \& 5 \& 12 \& 5 \& 2 \& 4 \& 4 \& 5 \& 4 \& v <br>
\hline 1 \& \& 3 \& 2 \& \& 4 \& 4 \& \& \& \& \& 4 \& \& \& <br>
\hline

 

\hline 11 \& 3 \& 3 \& 2 \& 2 \& 4 \& 4 \& 3 \& 8 \& 6 \& 4 \& 3 \& 4 <br>
\hline

 $\mathbf{V}$ 

4 \& 9 \& 3 \& 2 \& 6 \& 4 \& 6 \& 4 \& 4 \& 7 \& 10 \& 6 \& $\mathbf{W}$

 

6 \& 6 \& $\frac{3}{3}$ \& $\frac{2}{4}$ \& $\frac{6}{5}$ \& $\frac{4}{5}$ \& 6 \& $\frac{4}{6}$ \& $\frac{4}{6}$ \& $\frac{7}{4}$ \& $\frac{10}{7}$ \& 6 \& 6 \& 5 <br>
$\mathbf{x}$

 

$\mathbf{6}$ \& $\frac{6}{9}$ \& $\frac{3}{5}$ \& $\frac{4}{6}$ \& $\frac{5}{4}$ \& $\frac{5}{9}$ \& $\frac{6}{7}$ \& $\frac{6}{1}$ \& $\frac{4}{8}$ \& $\frac{7}{5}$ \& $\frac{5}{7}$ \& $\frac{4}{7}$ \& $\frac{9}{2}$ \& $\mathbf{Y}$ <br>
\hline
\end{tabular} NOPQRSTUVWYYZ

N O P Q R S T U V W X Y Z

A b C D E F G H I J K L M A \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|l|l}
5 \& 3 \& 5 \& 5 \& 9 \& 8 \& 6 \& 5 \& 1 \& 2 \& 5 \& 5 \& 2 <br>
\hline \& \& \& <br>
\hline

 B 

9 \& 5 \& 6 \& 4 \& 8 \& 6 \& 3 \& 5 \& 6 \& 4 \& 7 \& 4 \& 6 <br>
\hline

 C 

4 <br>
4 <br>
2
\end{tabular}$\overline{4}$

 E \begin{tabular}{l|l|l|l|l|l|l|lllllllll}
$\mathbf{F}$ \& 4 \& 6 \& 4 \& 3 \& 4 \& 9 \& 5 \& 5 \& 2 \& 4 \& 3 \& 4 \& 3 <br>
\hline

 G 

\hline 5 \& 3 \& 4 \& 1 \& 6 \& 5 \& 6 \& 2 \& 4 \& 3 \& 4 \& 3 \& 5 <br>
\hline

 H 

\hline 3 \& 6 \& 4 \& 7 \& 2 \& 4 \& 5 \& 4 \& 3 \& 6 \& 7 \& 5 \& 4 <br>
\hline

 

I \& 5 \& 5 \& 2 \& 3 \& 4 \& 2 \& 5 \& 8 \& 5 \& 4 \& 4 \& 2 \& 2

 J $\begin{array}{lllllllllllllllll}3 & 5 & 4 & 4 & 2 & 3 & 6 & 5 & 8 & 5 & 1 & 4 & 4\end{array}$ к 5 

K \& 5 <br>
\hline \& 8 <br>
\& <br>
\hline

 L 

8 \& 8 \& 6 \& 4 \& $\frac{5}{3}$ \& $\frac{6}{8}$ \& $\frac{2}{7}$ \& - \& 3 \& 2 \& $\frac{5}{5}$ \& $\frac{2}{5}$ \& $\frac{6}{4}$ \& $\frac{3}{6}$ <br>
\hline
\end{tabular}



A B C D E F G H I J K L M $\mathbf{N}$\begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|l|}
\hline 4 \& 5 \& 2 \& 6 \& 2 \& 3 \& 5 \& 4 \& 6 \& 4 \& 1 \& 5 \& 2 <br>
\hline

 O 

\hline 5 <br>
\hline
\end{tabular}$\overline{5}$

 Q \begin{tabular}{llllllllllllllll}
\& 2 \& 3 \& 3 \& 5 \& 9 \& 6 \& 6 \& 4 \& 8 \& 3 \& 4 \& 3 \& 1 <br>
\hline

 $\mathbf{R}$ 

$\mathbf{S}$ \& 3 \& 6 \& 7 \& 11 \& 4 \& 0 \& 6 \& 5 \& 3 \& 5 \& 6 \& 4 \& 7

 

$\mathbf{T}$ \& 2 \& 2 \& 7 \& 4 \& 4 \& 4 \& 4 \& 1 \& 6 \& 7 \& 3 \& 7 \& 3
\end{tabular}

 v $\overline{7} \times \overline{8}$ w -13 6 $\mathbf{W}$ | $\mathbf{X}$ | $\frac{4}{\mathbf{Y}}$ | $\frac{3}{3}$ | $\frac{4}{7}$ | $\frac{6}{3}$ | $\frac{4}{7}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{4}{6}$ | $\frac{3}{5}$ | $\frac{7}{2}$ | $\frac{5}{12}$ | $\frac{2}{5}$ | $\frac{6}{3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{Y}$ | $\mathbf{x}$ |  |  |  |  |  |  |  |  |  |  |  |  |




| 3 | 3 | 8 | 4 | 5 | 4 | 3 | 3 | 8 | 6 | 5 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A |  |  |  |  |  |  |  |  |  |  |  |  | | 7 | 1 | 7 | 4 | 5 | 3 | 3 | 0 | 2 | 4 | 7 | 5 | 5 | B |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | | 4 | 4 | 5 | 3 | 6 | 4 | 7 | 2 | 4 | 3 | 4 | 6 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $C$ |  |  |  |  |  |  |  |  |  |  |  |  | | 1 | 5 | 8 | 7 | 5 | 3 | 8 | 1 | 2 | 7 | 5 | 11 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



 $\overline{3}-\frac{1}{6}\left|\frac{8}{1}\right| \frac{5}{6}\left|\frac{4}{8} \frac{5}{11}\right| \frac{7}{4} \frac{6}{3}\left|\frac{2}{5}-\frac{1}{6}\right| \frac{7}{7}\left|\frac{3}{6}\right| \frac{3}{3}$ G $\mathbf{H}$ I | 4 | 3 | 2 | 1 | 8 | 2 | 5 | 6 | 5 | 5 | 7 | 6 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I |  |  |  |  |  |  |  |  |  |  |  |  |

 $\begin{array}{llllllllllllllllll}\mathbf{K} & 0 & 6 & 4 & 6 & 3 & 7 & 6 & 2 & 5 & 3 & 3 & 6 & 5 & \mathrm{~K}\end{array}$ | $\mathbf{L}$ | 3 | 3 | 2 | 1 | 6 | 3 | 0 | 2 | 5 | 15 | 4 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{L}$ |  |  |  |  |  |  |  |  |  |  |  |

 N O P \& R S T U V W X Y Z \begin{tabular}{|l|l|l|l|l|l|l|l|l|l|l|l|l|}
\hline 8 \& 4 \& 1 \& 4 \& 4 \& 3 \& 6 \& 1 \& 2 \& 4 \& 4 \& 3 \& 7 <br>
$\mathbf{N}$

 

\hline 4 \& 2 \& 4 \& 3 \& 2 \& 6 \& 3 \& 5 \& 4 \& 4 \& 3 \& 5 \& 6 \& O

 $\begin{array}{llllllllllllllll}3 & 6 & 1 & 4 & 3 & 2 & 5 & 3 & 6 & 3 & 3 & 5 & 3 & P\end{array}$ 

\hline 5 \& 2 \& 2 \& 6 \& 3 \& 6 \& 6 \& 3 \& 4 \& 5 \& 7 \& 1 \& 5 \& $Q$

 

\hline 8 \& 5 \& 7 \& 2 \& 6 \& 9 \& 4 \& 4 \& 5 \& 4 \& 6 \& 7 \& 4 \& $R$
\end{tabular}

 | 5 | 7 | 8 | 6 | 6 | 5 | 6 | 10 | 10 | 6 | 4 | 7 | 5 | $T$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

 \begin{tabular}{|l|l|lllllllllllll|l}
\hline 2 \& 6 \& 4 \& 1 \& 4 \& 5 \& 7 \& 1 \& 1 \& 4 \& 4 \& 7 \& 7 \& v

 

\hline 1 \& 5 \& 4 \& 2 \& 4 \& 4 \& 4 \& 6 \& 2 \& 3 \& 5 \& 6 \& 4 \& W

 

1 \& 4 \& 7 \& 6 \& 8 \& 7 \& 5 \& 7 \& 4 \& 7 \& 4 \& 5 \& 7 \& $\mathbf{x}$

 

\hline 1 \& 4 \& 5 \& 6 \& 5 \& 9 \& 8 \& 11 \& 1 \& 5 \& 3 \& 4 \& 9 \& $\mathbf{Y}$
\end{tabular}

 N OPQRSTUVWXYZ

.


[^0]:    Result: In AL4: Place H under K of NCAL4

[^1]:    In locus $U d, O_{\mathrm{D}}=\mathrm{H}_{\mathrm{c}}$ and in locus $N^{\prime} d, \mathrm{O}_{\mathrm{D}}=\mathrm{K}_{\mathrm{c}}$
    In locus $U e, \mathrm{E}_{\mathrm{p}}=H_{\mathrm{c}}$ and in locus $N^{\prime} e, \mathrm{E}_{\mathrm{p}}=\mathrm{L}$

